CHAPTER 1

Some necessary information related to astronomy and history of astronomy

1. THE ECLIPTIC. THE EQUATOR. PRECESSION

Let us consider the motion of the Earth along its solar orbit. It is usually considered that it isn't the Earth itself that rotates around the Sun, but rather the mass centre (gravity centre) of the Earth-Moon system, or the so-called barycentre. The barycentre is relatively close to the centre of the Earth as compared to the distance between the Earth and the Sun. The stipulations of the present work allow us to consider the orbital motion of the barycentre around the Sun identical to the orbital motion of the Earth itself.

Gravitational perturbations caused by planets cause constant rotation of the barycentre orbit plane. This rotation contains a certain primary sinusoidal compound with very high periodicity. It is complemented by certain minor variable fluctuations, which we shall ignore. This rotating orbital plane of the Earth is called the ecliptic plane.

Sometimes the term "ecliptic" is used for referring to the circumference where the ecliptic plane crosses the imaginary sphere of immobile stars. Let us assume that the centre of this sphere coincides with the centre of the Earth that lies on the ecliptic plane. In fig. 1.1. it is indicated as point *O*. We can disregard the motion of the Earth in relation to the distant stars and consider it the immobile centre of the stellar sphere. Our further references to celestial objects such as the Sun, stars etc shall imply the identification of said object with the point of its projection over the sphere of immobile stars.

The ecliptic rotates with time, which is why it is known as the "mobile ecliptic". In order to refer to the position of the mobile ecliptic at a given point in time, let us introduce the concept of instantaneous ecliptic for a given year or epoch. The conception and the properties of instantaneous spin vector pertain to the discipline of celestial mechanics. Fixed successive instantaneous ecliptics for different epochs are sometimes referred to as fixed ecliptics of said epochs. For instance, it is convenient to refer to the fixed ecliptic for 1 January 1900. The position of the mobile ecliptic for any given point in time can be specified in relation to a randomly chosen fixed ecliptic.

The Earth is considered a perfectly solid body in celestial mechanics. It is well known that a solid body possesses a so-called inertia ellipsoid, which is rigidly defined by its three semi-axes. The rotation of a solid body is characterised by the value and the spatial attitude of spin vector ω. Vector ω is sometimes referred to as the instantaneous axis of rotation. The semi-axes of the inertia ellipsoid are orthogonal, and

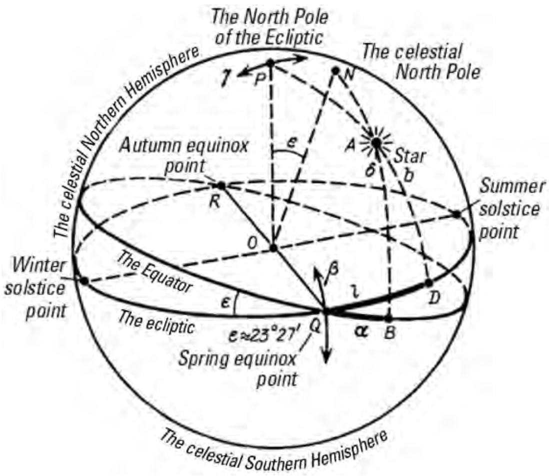

Fig. 1.1. The sphere of immobile stars. The ecliptic and equatorial coordinate systems.

can therefore be used as an orthogonal system of coordinates. Thus, vector ω can be defined by the projections of *x*, *y* and *z* over the axes of inertia. The moments of body inertia relative to these axes shall be indicated as *A, B* and *C*, respectively. The rotation of a solid body is described in the dynamic equations of Euler-Poisson:

$$A\dot{x} + (C - B)yz = M_A$$
$$B\dot{y} + (A - C)xz = M_B$$
$$C\dot{z} + (B - A)xy = M_C$$

In the right part of the equations we have the projections of vector *M*, known as the external couple in relation to the mass centre, over the same axes. Moment *M* results from the effect of solar and lunar gravity on the ellipsoidal figure of the Earth. The Earth is usually considered a two-axial ellipsoid rather than triaxial – an ellipsoid of revolution, in other words.

The position of vector *M* in relation to the axes of inertia changes rapidly, and these changes are of a rather complex nature; however, the application of modern theories of lunar and telluric motion makes it feasible to calculate its evolution with sufficient precision for any moment in time. This allows us to solve the equation of Euler-Poisson, or calculate the evolution of vector ω.

The "Tables of the Motion of the Earth on its Axis and Around the Sun" ([1295]) compiled by the eminent American astronomer Simon Newcomb are used in order to account for all the irregularities inherent in the motion of the Earth.

The study of cases (solid body configurations) when the equations of Euler-Poisson can be solved with sufficient precision comprises an important area of modern theoretical mechanics, physics and geometry.

Let us consider vector ω of instantaneous Earth rotation. It defines the axis of rotation, or the instantaneous rotation axis. The points where it crosses the surface of the Earth are known as instantaneous poles of the Earth, whereas those where it crosses the celestial sphere, or the sphere of immobile stars, are known as celestial poles (North and South). Let us consider the plane orthogonal to the instantaneous rotation axis of the Earth that crosses the mass centre of the Earth. Its intersection with the surface of the Earth is known as the instantaneous equator of Earth rotation, and the intersection with the celestial sphere is referred to as the true celestial equator, celestial equator or equinoctial.

Fig. 1.1 depicts the celestial sphere. Its centre is marked *O*. *P* stands for the North Pole of the ecliptic, and *N* – for the celestial pole. The ecliptic and the equator have two intersection points, which are known as the vernal and autumnal equinox points (indicated as *Q* and *R* in fig. 1.1, respectively). The illustration also demonstrates the alterations of the star's coordi-

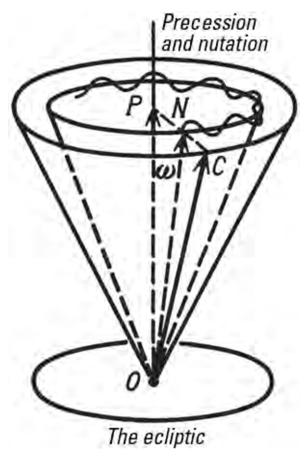

Fig. 1.2. Precession and nutation.

nates in relation to the two coordinate systems of the celestial sphere – equatorial and ecliptic.

Let us now consider a coordinate system that would not rotate together with the Earth, but be based on the ecliptic instead. The new coordinate system does not have to be orthogonal. The following axes are normally used for such coordinate systems:

1) normal to the ecliptic plane;

2) the intersection axis of the ecliptic and equatorial planes, or the equinoctial axis;

3) inertia axis *C*.

The projections of instantaneous angular velocity vector ω over these three axes are indicated as $\dot{\psi}$, $\dot{\theta}$ and $\dot{\varphi}$. We have thus expanded the Earth rotation rate into three components. What is their geometrical meaning? The value of $\dot{\psi}$, is known as the Earth precession rate. This component defines the circular conical motion of precession axis *C*, or the third axis of inertia, around the normal *OP*, as shown in fig. 1.2. Vector ω = *ON* follows this conical rotation. Let us point out the close proximity of vectors ω and *OC*. For approximated calculations we can assume vector ω to coincide with axis *OC*.

Owing to precession, the equinox axis, or the intersection of the ecliptic and the equator, rotates within the ecliptic plane. The rotation of $\dot{\theta}$ affects the inclination of axis *OC* towards the ecliptic to a certain extent. Finally, the value of $\dot{\varphi}$ defines the rate of the Earth's rotation around axis *OC*. In theoretical mechanics the value of $\dot{\varphi}$ is known as proper rotation rate. It is much higher than the angular velocities of $\dot{\psi}$ and $\dot{\theta}$. From the point of view of theoretical mechanics, this circumstance reflects the fact that the stable rotation of a solid body occurs around the axis that happens to be the closest to the axis of maximal inertia moment, or the shortest axis of the inertia ellipsoid. Let us remind the reader that the Earth is somewhat flattened at the poles.

Thus, $\omega = \dot{\psi} + \dot{\theta} + \dot{\varphi}$ (+ standing for the summation of vectors). Each velocity ($\dot{\psi}$, $\dot{\theta}$ and $\dot{\varphi}$) contains a single constant (or nearly constant) component as well as a great many small periodic ones, commonly referred to as nutations. If we overlook them, we shall come up with the following model of Earth rotation.

1. Constant velocity component $\dot{\psi}$ is called longitudinal precession. It moves axis *OC* along the circular cone with the approximate annual velocity of 50" (see fig. 1.2). The equinoctial axis moves clockwise along the ecliptic as seen from the side of its north pole. The precession vector is directed at the ecliptic's South Pole.

2. Constant velocity component $\dot{\theta}$ approximates 0.5" per year as of today.

3. Constant velocity component $\dot{\varphi}$ is the average proper Earth motion velocity value with the periodicity of one day anticlockwise around axis *AC* (as seen from the North Pole of the Earth).

Let us note that axis *OP*, which is the normal towards the ecliptic plane, belongs to the same plane as vector ω, which represents the instantaneous angle velocity of the Earth, and axis *OC*, or the third axis of inertia. This plane rotates around axis *OP* due to precession.

Nutational components inherent in velocities ($\dot{\psi}$, $\dot{\theta}$ and $\dot{\varphi}$) distort the above model – therefore, vector ω doesn't follow an ideal conical trajectory, but a rather erratic wavy one instead, which approximates the shape of a cone. The trajectory of the vector's end point is drawn as a wavy line in fig. 1.2.

The two circumferences that pertain to the celestial sphere (the equator and the ecliptic) intersect at the angle of ε = +23°27' in two points – *Q* and *R*, qv in fig. 1.1. The Sun crosses the equator twice in these points over the course of its annual voyage along the ecliptic. Point *Q*, which is where the Sun enters the Northern Hemisphere, is the point of the vernal equinox. This is the point where the respective durations of daytime and night time equal one another everywhere on the Earth. Point *R* corresponds to the autumnal equinox (see fig. 1.1).

The mobile ecliptic is in constant rotation. Therefore, the vernal equinox point constantly shifts alongside the equator, simultaneously moving along the ecliptic as well. The velocity at which the equinox point travels along the ecliptic is the actual longitudinal precession. The shift of the equinox points produces the equinox precession effect (see fig. 1.1).

2. EQUATORIAL AND ECLIPTIC COORDINATES

In order to record the observations of celestial bodies, one needs a convenient coordinate system that would allow one to fix the respective positions of celestial bodies. There are several such coordinate systems – first and foremost, the equatorial coordinates, which are defined as follows.

In fig. 1.1 we see the North Pole indicated as *N* and the celestial equator, which contains arc *QB*. We can estimate the plane of the celestial equator to coincide with the plane of the Earth equator, given that the centre of the Earth corresponds to point *O*, which stands for the centre of the celestial sphere. Point *Q* is the vernal equinox point. Let point *A* represent a random immobile star. Let us consider meridian *NB*, which crosses the North Pole and star *A*. Point *B* is the intersection of the meridian with the equatorial plane. Arc $QB = \alpha$ corresponds to the equatorial longitude of star *A*. This longitude is also known as "direct ascension". The direction of the arc is opposite to the motion of *Q*, which is the vernal equinox point. Therefore, direct ascensions of stars attain greater values over the course of time due to precession.

Meridian arc $AB = \delta$ corresponds to the equatorial latitude of star *A*, which is also referred to as the declination of star *A*. If we are to disregard the fluctuations of the ecliptic, the declinations of the stars located in the Northern Hemisphere diminish with time due to the motion of vernal equinox point *Q*. The declinations of the stars in the Southern Hemisphere slowly grow with time.

The daily motion of the Earth does not alter the declinations of the stars. Direct ascensions change in a uniform fashion and are affected by the Earth's rotation velocity.

The ecliptic coordinate is also rather popular, and it was used very widely in the ancient star catalogues.

Let us consider the celestial meridian that crosses the ecliptic pole *P* and star *A* (see fig. 1.1). It crosses the ecliptic plane in point *D*. Arc *QD* corresponds to ecliptic longitude *l* in fig. 1.1, and arc *AD* represents ecliptic latitude *b*. Precession makes arc *QD* grow by circa one degree every 70 years, which results in the uniform growth of the ecliptic longitudes.

If we disregard the fluctuations of the ecliptic, we can consider ecliptic latitudes *b* stable as a first approximation. This is the very thing that made ecliptic coordinates so popular with the mediaeval astronomers. The advantage of the ecliptic coordinates over the equatorial ones is that the value of *b* is constant, whereas the value of *l* grows with the course of time as a result of precession. The alterations of equatorial coordinates caused by precession conform to much more complex formulae, which account for the orthogonal turn of the ecliptic that connects it to the equator.

It is for this very reason that mediaeval astronomers tried to compile their catalogues with the use of ecliptic coordinates, notwithstanding that equatorial coordinates are easier to calculate by observations, since such calculations do not stipulate to define the ecliptic plane. The position of the ecliptic depends on the motion of the Earth around the Sun and requires the use of sophisticated methods for its calculation, which, it turn, lead to additional systematic errata in the coordinates of all stars. The discovery of the fact that the ecliptic fluctuates over the course of time led to the use of equatorial star coordinates in catalogues instead of the ecliptic system. This system is still used – the "advantage" of the ecliptic system is a thing of the past.

3. THE METHODS OF MEASURING EQUATORIAL AND ECLIPTIC COORDINATES

Let us briefly consider a number of actual methods used for the estimation of equatorial and ecliptic coordinates. We shall relate a certain simple geometric idea that such measuring instruments as the sextant, the quadrant and the transit circle employ in their construction.

Let us assume that observer *H* is located in point φ on the surface of the Earth (see figs. 1.3 and 1.4). It is rather easy to define line HN' that is oriented at the celestial North Pole and the parallel line *ON*. Next we have to define the meridian that crosses point *H* and mount a vertical wall on Earth surface that shall go along this meridian, qv in figs. 1.3 and 1.4. Marking the direction of the celestial pole on this wall as HN', we can also indicate the equatorial like HK', which is parallel to *OK*, by means of laying an angle $\frac{\pi}{2}$ from direction HN'. Right angle $N'HK'$ can be divided into degrees, which gives us an astronomical instrument for angular measurements – a quarter of a divided circle positioned vertically. Modern meridian instruments are based on this instrument as well – it can be used for measuring star declinations, or their equatorial latitudes, and also for marking the mo-

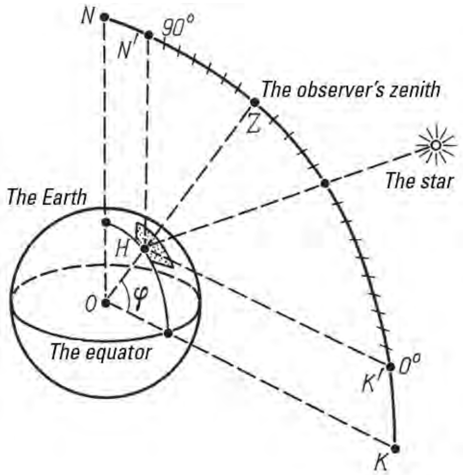

Fig. 1.3. The principle of stellar coordinate measurement.

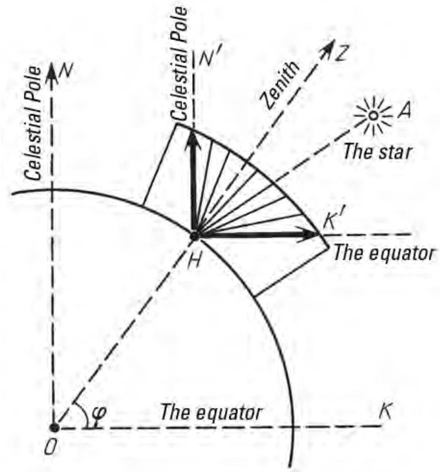

Fig. 1.4. Measuring the coordinates of a star that passes a meridian.

ments when stars cross a given meridian, or the so-called vertical.

A series of independent consecutive measurements makes it feasible to estimate the equatorial plane for the latitude of observation with high enough precision. At the same time, as it is obvious from the above elementary celestial mechanics, a measurement of longitudes requires a fixation of moments when the stars cross the meridian. This requires either a sufficiently precise chronometer, or an auxiliary device providing for fast measurements of longitudinal distances between the star that interests us and a fixed meridian. At any rate, longitudinal measurements are a substantially more subtle operation. Therefore it is to be expected that mediaeval astronomers' measurements of direct ascensions are cruder than their declination measurements.

In order to measure the ecliptic coordinates of stars observer *H* must assess the celestial position of the ecliptic first. This operation is sophisticated enough and stipulates a good understanding of primary elements of solar and telluric motion. Ancient methods of measuring the declination angle between the ecliptic and the equator as well as the position of the equinoctial axis with the aid of the armillary sphere or the astrolabe are described in [614] and a wealth of other sources. It has to be noted that in order to measure the ecliptic coordinates of a series of stars one needs a timekeeping device of some sort in order to compensate the daily rotation of the Earth and keep the orientation at the equinoctial point constant.

The obvious complexity of this task led to the following: for actual calculations of ecliptic coordinates astronomers would either use formulae of the celestial sphere's rotation or celestial globes with equatorial and ecliptic coordinate grids. The knowledge of equatorial coordinates would allow calculating their ecliptic equivalents. Naturally enough, there were inevitable errata resulting from lack of sufficient precision in the estimation of the comparative positions of the ecliptic and the equator, as well as the attitude of the equinoctial axis.

This very concise discussion of methods used for the measurement of ecliptic coordinates permits the estimation that the mediaeval astronomers are most likely to have used the following algorithm:

1) They would calculate the equatorial coordinates, the latitudinal measurements being more precise than the longitudinal.

2) Next they would estimate the position of the ecliptic and the equinoctial axis in relation to the equator.

3) Finally they would convert the equatorial coordinates into their ecliptic equivalents with the aid of special measurement instruments or trigonomet-

ric formulae (or, alternatively, with the use of a celestial globe with a double coordinate grid).

Moreover, since all the ancient measurement tools were inevitably installed upon the surface of the Earth, the above algorithm is the only real method of calculating the ecliptic stellar coordinates. Since a measuring instrument installed on the surface of the Earth takes part in daily rotation of the Earth, the instrument in question is invariably tied to the equatorial coordinate system.

The application of our statistical methods to the data provided by the Almagest catalogue yielded a confirmation of the above algorithm's usage, as we shall demonstrate below.

4. THE MODERN CELESTIAL SPHERE

In order to date an old star catalogue by the numeric values of stellar coordinates contained therein, we must be able to calculate the positions of stars on the celestial sphere for various points of time in the past. The information that we use for reference is the existing description of the celestial sphere in its modern state. The only data of importance are the coordinates of stars, as well as their magnitude and proper motion rate.

Jumping ahead, we can remark that the dating method that we suggest is only applicable if the respective positions of stars alter with the course of time. The rotation of the entire celestial sphere resulting from a transition to another coordinate system cannot be used for the purposes of independent dating. We shall discuss this in more detail below.

Let us discuss the characteristics of the stars that we shall refer to in our research.

The magnitude of a star in a modern catalogue is the number that represents its brightness. The lower the value, the brighter the star. There is an old tradition of indicating said values in star catalogues. The Almagest contains the magnitude values of all the stars it lists. The brightest stars are indicated as the stars of the first magnitude, the less bright ones correspond to the second magnitude and so on. Modern catalogues use the same scale for referring to the brightness of a given stars. However, stellar magnitudes can also be expressed as fractions. For example, Arcturus, which possesses the magnitude of 1 in the Almagest, has the magnitude of 0.24 in "The Bright Star Catalogue", a modern source ([1197]), and Sirius, also a star of the first magnitude in the Almagest, possesses the magnitude of –1.6 in the modern catalogue. Thus, Sirius is brighter than Arcturus, although Ptolemy believed them to be equally bright.

The matter might be that in the antiquity the brightness (or the magnitude) of a star was estimated by the observer in a very approximated fashion. Nowadays stellar magnitude is estimated with the photometric method. A comparison of stellar magnitudes contained in the Almagest to their modern precise values as given in the work of Peters and Knobel ([1339]) demonstrates that the discrepancy doesn't usually exceed 1 or 2 measurement units.

In our calculations of actual positions of stars in the past we were primarily referring to the bright star catalogue ([1197]), which contains the characteristics of circa 9000 stars up to the eighth stellar magnitude. Let us remind the reader that one can only see the stars whose magnitude is up to 6 or 7 with the naked eye. According to Ptolemy's claim, the Almagest star catalogue contains all the stars from the visible part of the sky up to the 6th magnitude.

Ptolemy was exaggerating – there are more stars with magnitudes of 6 and less in the visible part of the sky than in the Almagest catalogue. This is one of the reasons why the attempts to identify the Almagest stars with the stellar positions calculated "in reverse" lead to ambiguities (see Chapter 2 for more details). On the other hand, it would be natural to assume that all the stars that were actually observed by Ptolemy or his predecessors still exist and can be found in the modern catalogue ([1197]).

J. Bayer, a prominent XVII century astronomer, suggested a new system of referring to stars in a constellation. He suggested using letters of the Greek alphabet instead of a verbal description of a given star's position in a constellation. The brightest star of a constellation would be indicated by letter α, the second brightest one – by letter β, and so on. Later on, Flamsteed (1646-1720) devised a special numeration for stars in a constellation – more specifically, the westernmost star of a constellation was indexed as 1, the next one to the east – as 2, and so on. Flamsteed's numbers and Bayer's letters are often used in combination for referring to a star (32 α Leo and so on). Apart from that, some of the stars have individual names. Such "named" stars are comparatively rare – individual names were only assigned to stars that had special significance in ancient astronomy. For instance, 32 α Leo is called Regulus.

We have used the following characteristics of stars from the modern catalogue ([1197]):

1. *Direct ascension of a star* for the epoch of 1900, which is transcribed as α_{1900} below, expressed in hours, minutes and seconds.

2. *The declination of a star* for the same epoch transcribed as δ_{1900} and measured in degrees, arc minutes and seconds.

3. *Stellar magnitude*.

4. *Proper motion rate of a given star*. The proper motion rate is comprised of two elements, the first one being the star declination fluctuation rate and the second – the rate of its direct ascension alteration. However, the coordinate grid of longitudes and latitudes on a sphere isn't uniform. The distances between adjacent meridians diminish closer to the poles; therefore, the stellar velocity component of direct ascension gives one a wrong idea of the true, or "visible" velocity of a star on the celestial sphere in the direction of the parallel. Therefore, some modern star catalogues give the stellar velocity component of the direct ascension reduced to the equator. This means the value is multiplied by the declination cosine, which makes it possible to interpret it as the local Euclidean length of the stellar velocity vector projection over the equator (the parallel). This permits a comparison of the first stellar velocity components regardless of their proximity to the pole. If the velocities aren't reduced in this fashion, such comparisons require additional calculations.

Catalogues BS4 ([1197]) and BS5 (online source) that we have used, the velocities are reduced to the equator, which isn't the case with catalogues FK4 ([1144]) and FK5 (online source). Oddly enough, this fact isn't always mentioned in the descriptions of astronomical catalogues. The form of the direct ascension velocities has to be estimated from their actual numeric values.

The values of proper star motion rates are rather small. They don't normally exceed 1" per year – the fastest of the stars visible to the naked eye, such as o^2 Eri, μ Cas, move at the rate of 4" per year.

The trajectories of stellar motion for the time intervals that interest us (2-3 thousand years) can be considered straight, which means that each of the star's coordinates on the celestial sphere change evenly. This approximation is only valid for areas that lay at some distance from the pole, obviously enough.

The standard coordinate system for the celestial sphere as given in the modern star catalogues is customarily based on the equatorial coordinates for the epochs of 1900, 1950 and 2000 A.D. We have chosen the system of equatorial coordinates for the beginning of 1900 A.D. Further calculations and coordinate system conversions for a given epoch *t* were based on this system.

First and foremost, in order to date the Almagest catalogue we shall need the coordinates of stars with high proper motion rates. Naturally, we shall only consider the fast stars that are believed to be listed in the Almagest.

We have refrained from discussing the issue of whether or not the Almagest stars were identified correctly. We shall consider it in detail below. In order to solve the identification problem we must know whether a given star had an individual name in the ancient catalogues. The information about the mediaeval names of stars was taken from catalogues BS4 ([1197]) and BS5 (online source).

In order to date the Almagest catalogue by proper motion rates we shall require the following two lists of stars from the modern catalogues. We shall merely describe them herein; the actual lists can be found in Annex 1.

We shall refer to the first list as to the list of "fast" stars. In the first stage of said list's compilation we have selected all the stars whose speed by one of the coordinates at least is greater than 0.1" per year. This list was subsequently reduced to the stars that either have Bayer's Greek letter or Flamsteed's number in their name. Thus, we have rejected the stars that are a priori useless for the dating for the Almagest. The matter is that nearly every star identified by the astronomers as one of the Almagest stars has an index in either Bayer's or Flamsteed's system, or both; also, if a star from the Almagest is identified as one that lacks such indices, this identification is always rather ambiguous ([1339]). The reason is clear enough. The catalogues of Bayer and Flamsteed were already compiled in the epoch of early telescopic observations, or the XVII-XVIII century. If a given star is omitted from those catalogues, it is either too dim or too difficult to tell apart from the celestial objects in its immediate vicinity.

There may be other complications in the same vein; therefore, one can hardly assume that a star of this sort can be veraciously identified as an Almagest star and that its position was measured with sufficient precision by the "ancient" astronomers.

The above selection gave us a list of "fast" stars visible with the naked eye, which can be found in modern star catalogues and identified as Almagest stars. Quite naturally, the veracity of such identifications requires a separate research. We shall consider this problem below.

Our list of "fast" stars visible to the naked eye can be found in Table P1.1 of Annex 1.

The second list of stars is the list of named stars. It is contained in Tables P1.2 and P1.3. In Table P1.2 the stars are arranged by names, and in Table P1.3 – by respective numbers from the Bright Star Catalogue ([1197]). This list contains all the stars which have individual names according to BS4 ([1197]), or which had such names in the past (Arcturus, Aldebaran, Sirius etc).

The lists of fast and named stars intersect – the same star can have a visible proper motion rate and an individual name. Such stars are the most useful for the dating of the Almagest.

5. "REVERSE CALCULATION" OF OBJECTS' POSITIONS ON THE CELESTIAL SPHERE. THE FORMULAE OF NEWCOMB-KINOSHITA

5.1. Necessary formulae

Having the modern coordinates and proper motion rates of stars at our disposal, we can compile a sufficiently precise star catalogue for any epoch in the past. By "sufficiently precise" we mean the precision that corresponds to modern astronomical theories, which is quite sufficient for our purposes. Such precision can be considered absolute in comparison to that of the old catalogues.

We had to perform retroactive star position calculations quite a few times for different epochs. We would first calculate the positions of stars on the celestial sphere for year t in coordinates α_{1900} and δ_{1900}, and then convert those into ecliptic coordinates l_t and b_t for epoch t.

Let us cite the necessary formulae that allow the conversion of coordinates α_s and δ_s into coordinates l_{s0} and b_{s0} for any epochs s and s_0. These formulae account for precession and proper star motion. Said formulae, as well as fig. 1.5, which illustrates them, were taken from [1222]. They are based on Newcomb's theory as modified by Kinoshita. The actual coordinate conversion procedure is described in the next section (5.2). In these formulae time moments s_0 and s are counted backwards from the epoch of 2000 A.D. in Julian centuries, and $\theta = s_0 - s$. See fig. 1.5.

$$\begin{aligned}\varphi(s,s_0) &= 174°52'27{,}66'' + 3289{,}80023''s_0 + 0{,}576264''s_0^2 \\ &- (870{,}63478'' + 0{,}554988''s_0)\theta + 0{,}024578''\theta^2;\end{aligned} \quad (1.5.1)$$

$$\begin{aligned}\kappa(s,s_0) &= (47{,}0036'' - 0{,}06639''s_0 + 0{,}000569s_0^2)\theta \quad (1.5.2) \\ &+ (-0{,}03320'' + 0{,}000569''s_0)\theta^2 + 0{,}000050''\theta^3;\end{aligned}$$

$$\begin{aligned}\varepsilon_0(s,s_0) &= 23°26'21{,}47'' - 46{,}81559''s_0 \\ &- 0{,}000412''s_0^2 + 0{,}00183''s_0^3\end{aligned} \quad (1.5.3)$$

$$\varepsilon_1(s,s_0) = \varepsilon_0(s,s_0) + (0{,}05130'' - 0{,}009203''s_0)\theta^2 - 0{,}007734''\theta^3;$$

$$\begin{aligned}\varepsilon(s,s_0) &= \varepsilon_0(s,s_0) + (-46{,}8156'' - 0{,}00082''s_0 + 0{,}005489''s_0^2)\theta \\ &+ (-0{,}00041'' + 0{,}005490''s_0)\theta^2 + 0{,}001830''\theta^3;\end{aligned}$$

$$\begin{aligned}\psi(s,s_0) &= (5038{,}7802'' + 0{,}49254''s_0 - 0{,}000039''s_0^2)\theta \\ &+ (-1{,}05331'' - 0{,}001513''s_0)\theta^2 - 0{,}001530''\theta^3;\end{aligned}$$

$$\begin{aligned}\chi(s,s_0) &= (10{,}5567'' - 1{,}88692''s_0 - 0{,}000144''s_0^2)\theta \\ &+ (-2{,}38191'' - 0{,}001554''s_0)\theta^2 - 0{,}001661''\theta^3;\end{aligned}$$

$$\begin{aligned}\Psi(s,s_0) &= (5029{,}0946'' + 2{,}22280''s_0 + 0{,}000264''s_0^2)\theta \\ &+ (1{,}13157'' + 0{,}000212''s_0)\theta^2 + 0{,}000102''\theta^3.\end{aligned} \quad (1.5.4)$$

Let us however note that the discrepancies between the corollaries made according to the actual theory of Newcomb and its modification made by Kinoshita ([1222]) that we have used are of no consequence insofar as our purposes are concerned. For any time moment t of the historical interval under consideration (between 600 B.C. and 1900 A.D.) the discrepancies between the ecliptic coordinates of a star calculated according to Newcomb's theory and those obtained with the use of its modified version ([1222]) are negligibly small in comparison to the errors of the Almagest. We have used [1222], since it gives the formulae for precession compensation in a format convenient for computer calculations.

5.2. The algorithm for calculating past positions of stars

Let us provide a detailed description of the algorithm used for the calculation of star catalogue $K(t)$, which reflects the condition of the celestial sphere for year t with sufficient precision, according to Newcomb's theory. Here t is a randomly chosen epoch from the historical interval under consideration (namely, one between 600 B.C. and 1900 A.D.). Epoch t is calculated backwards into the past from the epoch of 1900 in Julian years, in other words, $t = 1$ corresponds to the epoch of 1800, $t = 10$ – to the epoch of 900 A.D., $t = 18$ – to 100 A.D., etc. The discrepancy of several days that results from the differences between the Julian and the Gregorian calendar, and leads to the situation where the epoch of 100 A.D., for instance, fails to coincide with the epoch of 1 January 100 A.D. is of no importance whatsoever.

The calculated star catalogues $K(t)$ shall serve us for comparison with the old catalogue under study (such as the Almagest) with different values of t. Here t shall stand for a random assumed dating of an old catalogue. Thus, calculated catalogues $K(t)$ must be transcribed in ecliptic coordinates for epoch t. As it has been pointed out, all known old catalogues are compiled in ecliptic coordinates, be it Ptolemy's Almagest or the catalogues of As-Sufi, Ulugbek, Copernicus, Tycho Brahe etc.

Let us assume that the modern equatorial coordinates of a star in a catalogue (such as [1197]) are $\alpha^0 = \alpha^0_{1900}$, $\delta^0 = \delta^0_{1900}$. These coordinates reflect the position of the star in question for 1900 A.D. in the spherical coordinate system, whose equator corresponds to the Earth's equator in 1900 A.D. The equator is defined by the plane that is orthogonal to the axis of the Earth's rotation. Let us remind the reader that this plane's position changes over the course of time. We have to calculate the coordinates l_t, b_t, or the spherical coordinates whose equator coincides with the ecliptic, or the plane of the Earth's rotation around the Sun for epoch t. We should do the following for this purpose.

Step 1. We have to calculate the star's coordinates $\alpha^0(t)$, $\delta^0(t)$ for time moment t in the equatorial coordinate system for 1900 A.D. Bear in mind that the position of the stars on the celestial sphere changes over the course of time in relation to any fixed system of coordinates. The required calculations of the star's position are based on the known proper motion rates v_α, v_δ of the star by each of the coordinates α_{1900}, δ_{1900} (see Table 4.1, columns 5 and 6). We shall come up with the following for non-reduced proper motion rates:

$$\alpha^0(t) = \alpha^0_{1900}(t) = \alpha^0 - v_\alpha \cdot t,$$
$$\delta^0(t) = \delta^0_{1900}(t) = \delta^0 - v_\delta \cdot t.$$

Indeed, we can consider the proper motion rates of each star by the coordinates α_{1900}, δ_{1900} to be constant. The minus in the formulae cited above results from the retroactive nature of calculations; the velocity rate symbols v_α, v_δ correspond to the normal flow of time.

Before we can actually use this formula, we have to convert all the source values into a single measurement system. For instance, we can measure $\alpha^0(t)$ and $\delta^0(t)$ in radians, and the velocities v_α, v_δ – in (rad ÷ year) · 10^{-2}.

Step 2. We have to shift from coordinates α_{1900}, δ_{1900} to coordinates l_{1900}, b_{1900}. We shall come up with coordinates $l^0(t)$, $b^0(t)$ of our star for the moment t in spherical coordinates based on the ecliptic of the epoch of 1900 A.D. This is what we get:

$$\begin{aligned} \sin b^0(t) &= -\sin\alpha^0(t)\cos\delta^0(t)\sin\varepsilon^0 + \sin\delta^0(t)\cos\varepsilon^0, \\ \tan l^0(t) &= \frac{\sin\alpha^0(t)\cos\delta^0(t)\cos\varepsilon^0 + \sin\delta^0(t)\sin\varepsilon^0}{\cos\alpha^0(t)\cos\delta^0(t)}, \qquad (1.5.5) \\ \varepsilon^0 &= 23°27'8{,}26''. \end{aligned}$$

These formulae permit an unequivocal reconstruction of the values of $\beta^0(t)$ and $\alpha^0(t)$, since $-90° < b^0(t) < 90°$ and $|l^0(t) - \alpha^0(t)|\ \tau\ 90°$. The value of ε^0 corresponds to the declination angle between the ecliptic of 1900 A.D. and the equator of 1900 A.D. We refer the reader to the formula of 1.5.3, where one has to let $s^0 = -1$ in order to make the transition between 2000 A.D. and 1900 A.D.

Step 3. We have to make a shift from coordinates l_{1900}, b_{1900} to the auxiliary coordinates l^1 and b^1, which are also tied to the ecliptic of 1900. However, they have a different longitudinal reference point, which coincides with the intersection of the ecliptic of 1900 A.D.

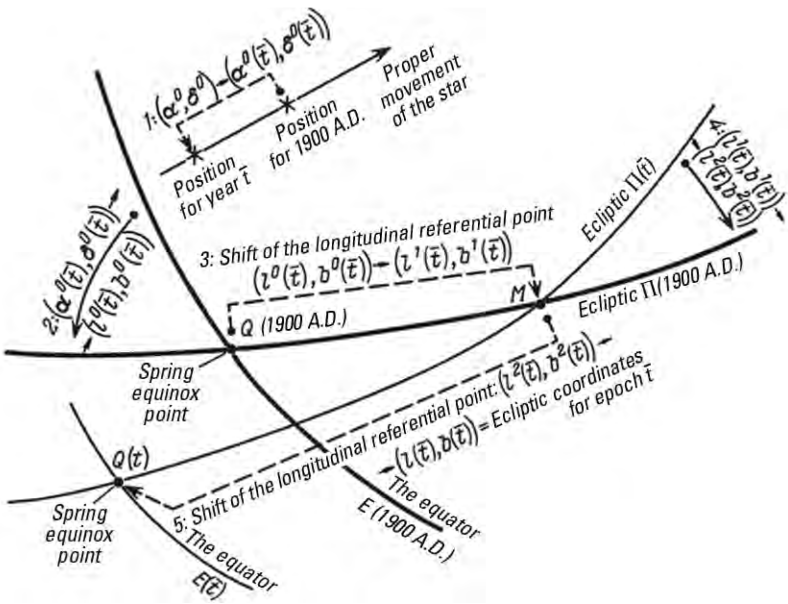

Fig. 1.6. The sequence of steps that we use for reverse calculations of stellar positions and their past coordinates.

and that of epoch t, or Π_{1900} and $\Pi(t)$. This transition conforms to the following formulae:

$$l^1(t) = l^0(t) - \varphi,$$
$$b^1(t) = b^0(t)$$
$$\varphi = 173°57'38.436'' + 870.0798''t + 0.024578''t^2. \quad (1.5.6)$$

Arc φ between the vernal equinox point of 1900 on the ecliptic Π_{1900} and the intersection of Π_{1900} and $\Pi(t)$ conforms to the formula (1.5.1) if we're to assume that $s_0 = -1$ and $\theta = -t$. Then the ecliptic $\Pi(s_0)$ in fig. 1.5 shall correspond to the ecliptic Π_{1900}. Ecliptic $\Pi(s)$ in fig. 1.5 shall represent the ecliptic of epoch t, which is of interest to us. Indeed, the time t is counted backwards from 1900 A.D. in centuries, whereas the remainder of $\theta = s - s_0$ is counted forwards from epoch s_0, also in centuries. Since we have agreed on $s_0 = -1$, which corresponds to 1900 A.D. (2000 – 100 = 1900), we have to choose $\theta = -t$ in order to make the epoch $s = s_0 + \theta$ correspond to epoch t under consideration in our formula (1.5.1).

Step 4. Next we have to make the transition from coordinates l^1, b^1 to coordinates l^2, b^2. These are spherical coordinates tied to the ecliptic $\Pi(t)$, whose only difference from the ecliptic coordinates l_t, b_t is due to the choice of the longitudinal reference point. In coordinates l^2, b^2 this point corresponds to the intersection of ecliptics Π_{1900} and $\Pi(t)$. The formulae of transition from l^1, b^1 to l^2, b^2 correspond to the formulae (1.5.5). Instead of ε^0 we have to take the angle ε^1 between ecliptics $\Pi(t)$ and Π_{1900}:

$$\varepsilon^1 = -47.0706''t - 0.033769''t^2 - 0.000050''t^3.$$

This expression is derived from the formula (1.5.2) where $s = -1$ and $\theta = -t$.

Step 5. Finally, we have to make the transition from coordinates l^2, b^2 to the ecliptic coordinates l_t, b_t. This transition conforms to the following formulae:

$$l_t = l^2 + \varphi + \Psi, \quad b_t = b^2,$$

where φ is defined in (1.5.6) and Ψ is defined by formula (1.5.4) with $s^0 = -1$ and $\theta = -t$, therefore

$$\Psi = -5026.872''t + 1.1314''t^2 + 0.0001''t^3.$$

The sequence of steps 1-5 as described above is illustrated in fig. 1.6.

Let us conclude by pointing out that all the calculations necessary for the dating of a given star catalogue can be performed without accounting for the Newcomb-Kinoshita theory. We shall consider this in more detail below. The Newcomb-Kinoshita theory is only used in order to obtain additional information concerning the errata in the estimation of the ecliptic plane made by the author of the catalogue. The value of these discrepancies is the auxiliary factor that confirms the correctness of our corollaries. See Chapters 6 and 7.

6. ASTROMETRY. ANCIENT ASTRONOMICAL MEASUREMENT INSTRUMENTS OF THE XV-XVII CENTURY

In Section 3 we have considered the general conception of angular measuring devices used in astronomy, which is important to us since it enables us to estimate the position of the equatorial line on the celestial sphere with sufficient precision.

Let us assume that the observer's line of eyesight is directed along half-line *HK'*, which moves along the line of the equinoctial in its daily rotation without any tergiversation. The attitude of half-line *HK'* will naturally depend on the geographical latitude. We can define the plane *HLM*, an orthogonal quadrant parallel to the equatorial plane, which crosses the celestial sphere precisely along the equinoctial, qv in fig. 1.7. It is therefore possible to construct a stationary device in said point of telluric surface, oriented by the north-south meridian, which allows marking the equator on the celestial sphere visually. This permits precise estimations of equatorial stellar latitudes – during their crossing of the quadrant's vertical plane, for instance. As we have already pointed out, the measurement of equatorial latitudes was hardly a complicated task for a professional astronomer of the XIV-XVI century. It required nothing but accuracy and sufficient time for observations. In particular, it has to be expected that a careful observer could not make a grave systematic error in the estimation of stellar declinations for a given year.

Now let us see how the simple general idea described above was implemented in real mediaeval instruments.

The first instrument is the meridian circle, or the so-called transit circle as described by Ptolemy (see fig. 1.8). The instrument looked like a flat metal ring of a random radius installed on a reliable support vertically in the plane of the local meridian. The circle was graded (into 360 degrees, for example). Another ring of a smaller diameter was placed inside the larger ring; it could rotate freely, remaining in the same plane as the larger ring (fig. 1.8). There are two little metallic plates with pointers attached to two opposing points on the inner ring (marked *P* in fig. 1.8); the pointers point at the grades found on the external ring. The device is installed in the plane of the local meridian with the aid of a level and the meridian line whose direction is defined by the shadow of a vertical pole at midday. Then the zero mark on the external ring of the instrument is synchronised with the local zenith.

The instrument described above can be used for measuring the height of the Sun at given latitude. One must quickly turn the inner ring at midday until the shadow of one plate *P* covers the other plate *P* completely. In this case, the position of the pointers on the plates shall tell us the height of the Sun with the aid of the grade marks on the external ring. It has to be pointed out that the instrument's indications are to be read after one fixes the plates in their proper positions. This tells one the height of the Sun already after midday. Moreover, the meridian circle can measure the angle between the ecliptic and the equator.

The second instrument is the astrolabon as described by Ptolemy, which is more frequently referred to as "astrolabe" in our days. The latter term is mediaeval in origin. According to the Scaligerian history of astronomy, the meaning of the term "astrolabon" has been changing over the course of time. We are told that "in deep antiquity", or around the very beginning of the new era, the term "astrolabon" was used for referring to the instrument that we shall describe shortly. Ptolemy used one of those. However, in the Middle Ages the instrument in question was already known as the armillary sphere, or "armilla". Some modern astronomers are of the opinion that Ptolemy describes the armillary sphere or the astrolabon in his "Almagest", and not the actual astrolabe (see [395], for instance). According to Robert Newton, a renowned astronomer, "it is likely that around the end of the Middle Ages the term 'astrolabe' referred

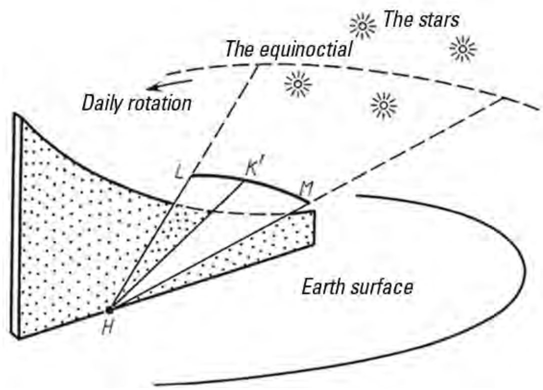

Fig. 1.7. Measuring the latitude of a star.

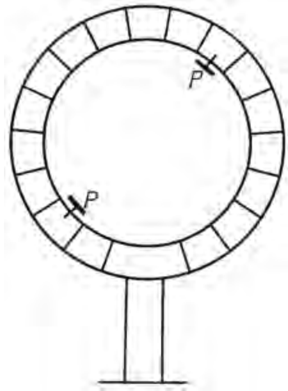

Fig. 1.8. The armillary circle.

to the device used for measuring the height of a celestial body above the horizon. As for the device we describe herein [in accordance with Ptolemy's indications – Auth.], by that time it was better known as the armillary sphere, which is the distant ancestor of the modern telescopes' bearings" ([614], page 151).

In order to avoid confusion with terms, we shall describe the two instruments separately – Ptolemy's astrolabon and the astrolabe, or the mediaeval instrument whose name is virtually identical to that of Ptolemy's astrolabon. The primary elements of the astrolabon's (armilla's) construction are shown in fig. 1.9. In fig. 1.10 we see the principal scheme of the mediaeval armillary sphere. Fig. 1.11 shows us "the mediaeval armillary sphere – of Ptolemy's type, according to historians. Its diameter equals 1.17 metres. This instrument was manufactured when Ptolemy's epoch was already considered ancient – it belonged to Tycho Brahe, the famed XVI century astronomer" ([1029], page 13). The implication is that astronomical instruments remained the same for fifteen hundred years. As we can see, the instruments of the "ancient" Ptolemy from the second century A.D. and the XVI century scientist Tycho Brahe were almost identical, as though they were made in the same mediaeval workshop. An ancient drawing of Tycho Brahe's large armillary sphere can be seen in fig. 1.12.

We must now describe the correct use of this instrument to the reader and also relate the astronomical principles of its construction. The main element of the armillary sphere comprises two metallic rings, perpendicular to one another and rigidly joined to-

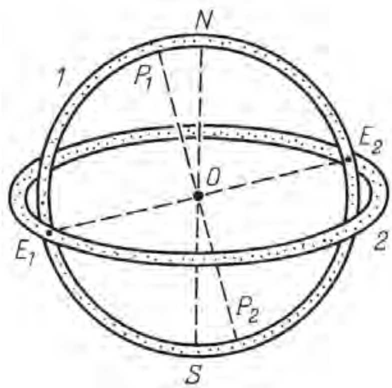

Fig. 1.9. A scheme of the astrolabon (armilla).

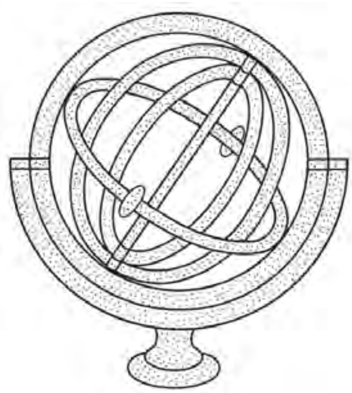

Fig. 1.10. A scheme of the armillary sphere.

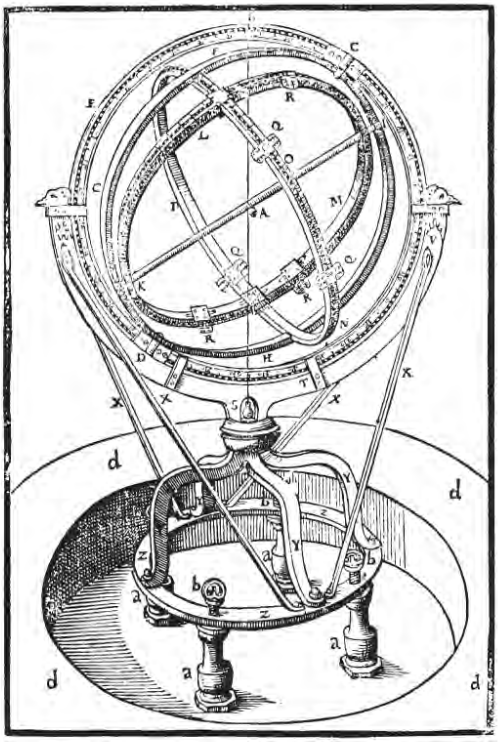

Fig. 1.11. The armillary sphere made in the XVI century; it used to belong to Tycho Brahe (1598). It is almost indistinguishable from the instrument used by the "ancient" Ptolemy in the II century A. D. These instruments are most likely to date to the same epoch – the XV-XVII century. Taken from [1029], page 13.

gether in points E_1 and E_2. Let us henceforth refer to the rings as the "first" and the "second" (see fig. 1.9). The first ring rotates around the axis *NS*, which is parallel to the axis of telluric rotation. The centre of both rings is point *O*; P_1P_2 is the perpendicular to the second ring's plane.

Let us describe how one uses the armilla in order to measure the angle between the ecliptic and the equator, for example. The most appropriate time for such measurements falls over the day of summer or winter solstice. The corresponding point on the orbit of the Earth is marked O' in fig. 1.13. It doesn't matter whether it corresponds to summer or winter solstice. Let us consider the plane that crosses the radial vector CO', where *C* is the Sun, and the Earth axis is indicated as NO'. Since O' is the solstice point, this plane will be orthogonal to the plane of the ecliptic, crossing the Earth surface along the meridian, qv in fig. 1.13.

Let us assume that the armilla is installed somewhere along this meridian. The instrument can be located anywhere on the surface of the Earth, but the measurements must begin at midday, which is when the instrument shall be on the meridian that is the intersection of said plane and the surface of the Earth. We assume the observer to know the direction of the Earth axis in this part of the Earth; therefore, the armilla's *NO* axis shall be oriented in this direction, parallel to axis NO', qv in fig. 1.13. Then, by rotating the first metallic ring around the armilla's axis *NS*, we shall install this ring in the plane of the meridian, which will happen when the shadow from the external edge of the ring shall cover the inner part of the ring exactly. Finally, having fixed the plane of the first ring, we must make the second ring orthogonal to the first, so that its inner part would be covered by the shadow cast by its outer part. Fig. 1.13 demonstrates that the second ring shall end up right in the plane of the ecliptic as a result of these manipulations (more precisely, it shall be parallel to the ecliptic plane). As we have fixed both rings in the necessary position, the perpendicular P_1P_2 to the second ring shall also be fixed, thus marking the pair of polar points P_1 and P_2 on the first ring. We shall therefore be able to measure the angle P_1ON with sufficient precision; it is obviously equal to the angle between the ecliptic and the equator.

We have described the method that was allegedly used by the ancient astronomers. Despite the geometrical simplicity of the idea, one can clearly see the numerous complications that introduce different errata into the numeric value of the measured angle. In particular, the observer must know the following parameters:

a) the direction of axis *ON*, which is parallel to the axis of the Earth;

b) the day of solstice;

c) the moment of midday in this point of Earth surface.

R. Newton made the following justified remark: "The primary shortcoming of this instrument is that one has to be rather quick when one uses it, since the rotation of the Earth has a negative effect on the precision of the device" ([614], page 150). Indeed, in fig. 1.13 we can see that the rotation of the Earth begins to turn the instrument around axis $O'N$, which renders the above considerations invalid.

Strictly speaking, the points O (the centre of the armilla) and O' (the centre of the Earth), as seen in fig. 1.13, are different points. The distance between the two is equal to the radius of the Earth. However, this discrepancy is negligibly small for the above calculations. Therefore, we can assume that $O = O'$ insofar as these measurements are concerned, as shown in fig. 1.13.

Let us come back to the measurements of the ecliptic coordinates with the aid of the armilla.

After the correct installation of the device as described above, it is tuned to the ecliptic coordinate system for a short time, namely, the plane of the second ring E_1E_2 is parallel to the ecliptic plane. Points E_1 and E_2 shall correspond to the solstice points. Both rings are presumed graded. Therefore, we can unambiguously define points R_1 and R_2 on the second ring, which shall correspond to the equinoxes. They divide arcs E_1 and E_2 in two halves. Points R_1 and R_2 are omitted from fig. 1.13 so as not to make the illustration too cluttered. Thus, what we have on the second ring is a scale with a fixed initial reference point (R_1, for instance, which is the vernal equinox point). We can thus measure ecliptic longitudes and latitudes of points on the celestial sphere, such as stars.

However, let us reiterate that the daily rotation of the Earth quickly sets off the precision of the instrument. Therefore, one needs a precise chronometer in order to compensate for the rotation of the Earth and tune the instrument. This is how the modern measurement instruments are constructed – the rotation of the Earth is compensated by the automatic tracking system.

In order to facilitate the measurements of celestial objects' ecliptic coordinates, a third ring is usually added to the armillary sphere – a rotating one. The axis of its rotation can, it turn, slide along the second ring, which is positioned in the plane of the ecliptic. We shall omit these details, since they are of little importance to us.

Let us now consider the third instrument, or the quadrant (see fig. 1.14). This instrument is based on the meridian circle and has a sharp pointer at its centre, which is perpendicular to the plane of this circle.

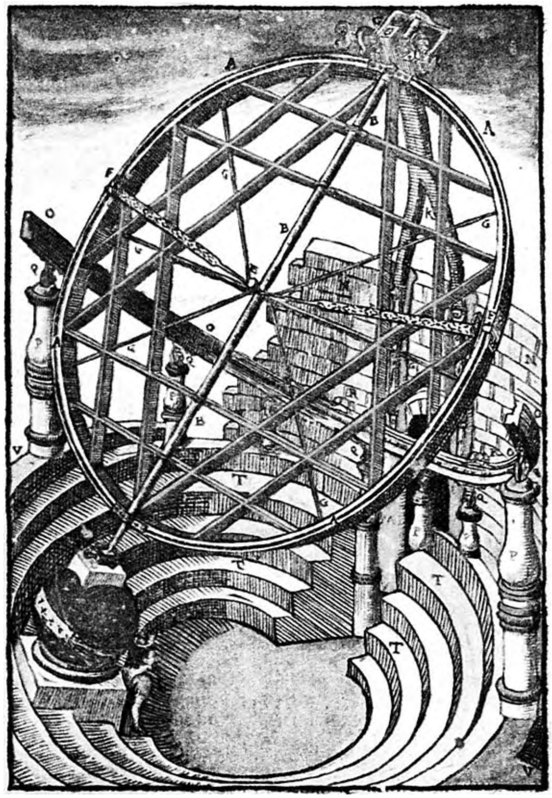

Fig. 1.12. "The large armillary sphere of Tycho Brahe for measuring the angular distances between luminaries" (from ***Mechanics Rejuvenated by Astronomy***, a work of Tycho Brahe. Windsbeck, 1598. Taken from [926], page 62.

The shadow from the pointer falls over the lower (northern) side of the meridian circle and can move within the confines of one quarter of the circumference. Therefore, in order to measure the height of the Sun it suffices to grade one quarter of the ring. The quadrant is therefore a plate of some sort with a graded quarter of a circle installed in the plane of the meridian. The height of the Sun above the horizon at midday is indicated by the shadow of the pointer that falls over the scale.

In fig. 1.15 we see the astronomical quadrant from a mediaeval book of 1542 by Oronce Fine ([1029], page 19).

Fig. 1.16 shows us a small quadrant with a radius of 39 centimetres, which belonged to Tycho Brahe ([1029], page 26).

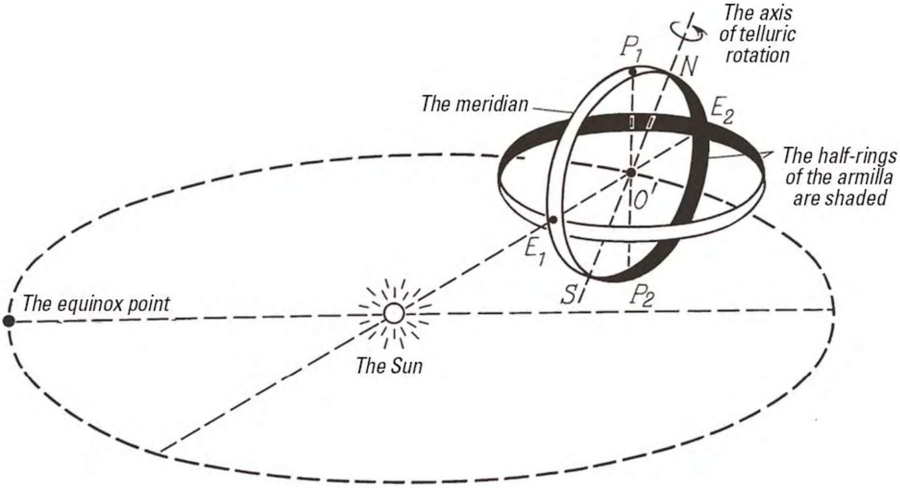

Fig. 1.13. A scheme of utilising the armilla for the measurement of the angle between the equator and the ecliptic, for instance.

In fig. 1.17 we see Tycho Brahe's sextant with a radius of 1.55 metres, and in fig. 1.18 – another sextant of Tycho Brahe of the same size ([1029], page 26).

In fig. 1.19 we see the astronomer Hevelius portrayed performing measurements with the aid of the sextant ([1029], page 67).

The fourth instrument is the astrolabe (see fig. 1.20). The mediaeval astrolabe is a round metallic plate with a diameter of some 50 centimetres, with a graded ring mounted rigidly on one of its edges. At the centre of the ring there is a mobile plank with visors mounted on an axis perpendicular to the centre of the circle. The instrument can be suspended vertically; there is a special loop at the edge of the plate that serves this purpose. The plane of the vertically suspended circle could be directed at a celestial body, likewise the rotating mobile plank. This is how the body's height above the horizon was measured. Apart from that, after the measurement of the height of the Sun at midday, one could also measure the observation latitude. The precision of such measurements must have been rather low due to the primitive nature of the method used. It is believed that the instrument in question could measure the observation

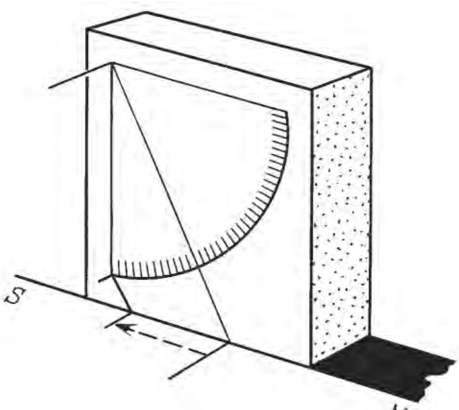

Fig. 1.14. A scheme of the quadrant.

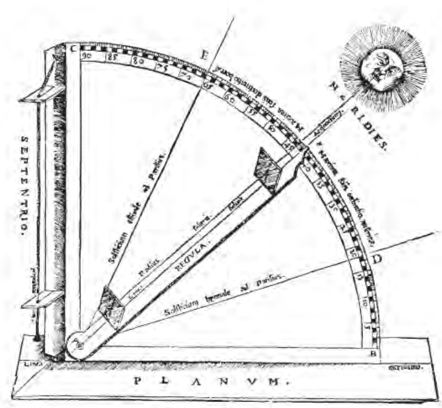

Fig. 1.15. An astronomical quadrant from a mediaeval book by Finney. Taken from [1029], page 19.

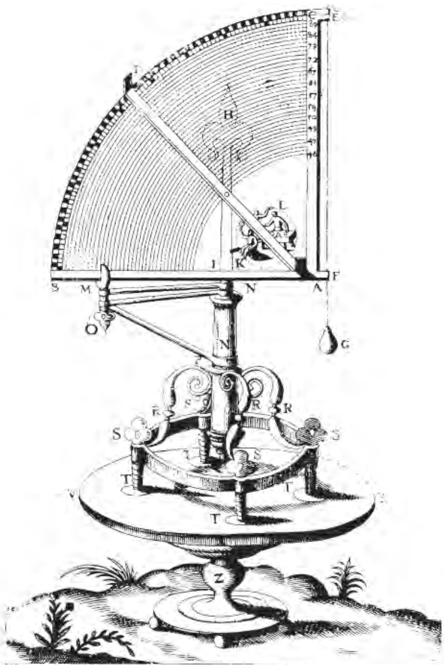

Fig. 1.16. A small quadrant of Tycho Brahe (1598). Taken from [1029], page 26.

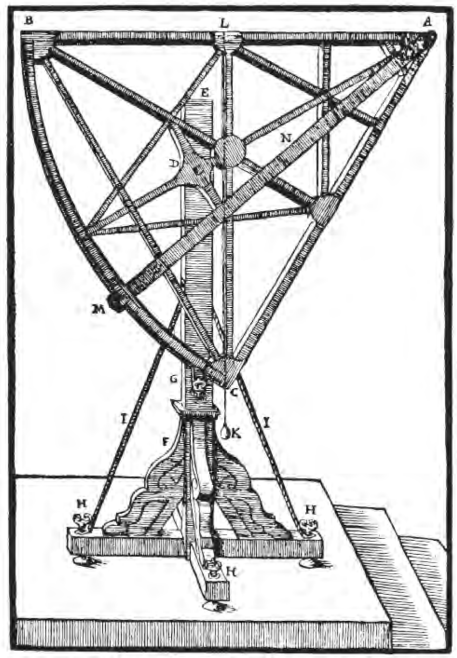

Fig. 1.17. The sextant of Tycho Brahe (1598). Taken from [1029], page 26.

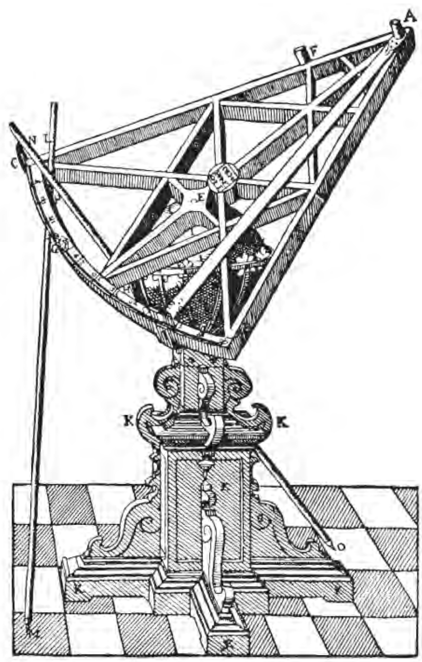

Fig. 1.18. Another sextant that belonged to Tycho Brahe (1598). Taken from [1029], page 26.

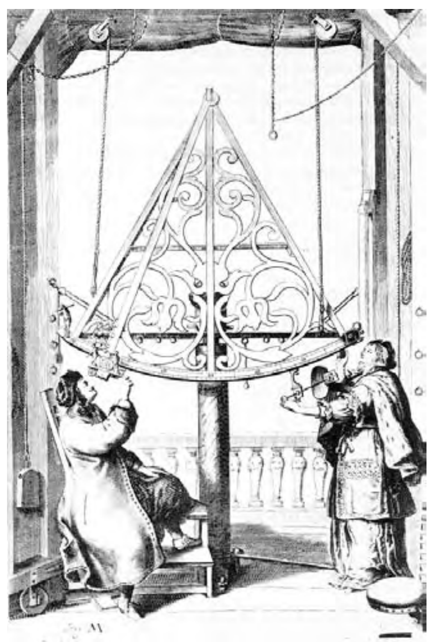

Fig. 1.19. The astronomer Hevelius is using a large sextant for observations, assisted by his wife. Ancient engraving dating to 1673. Taken from [1029], page 67.

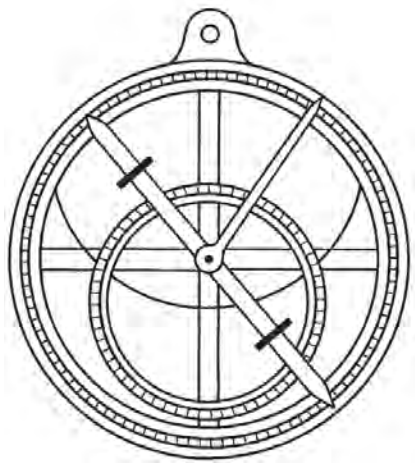

Fig. 1.20. A scheme of the astrolabe.

point latitude with the precision of several arc minutes ([614]).

In fig. 1.21 we see an old astrolabe of 1532 (Georg Hartmann, Nuremberg). We see the front and the reverse of the astrolabe.

In fig. 1.22 we reproduce an old picture of the famous mediaeval astronomical instrument known as "the Turkish tool", or "torquetum" ("turquetum"). Specialists in the history of science tell us the following: "The 'torquetum' (or 'turketum'), whose name can be translated as 'the Turkish tool', was characteristic for the mediaeval European astronomy, and embodies the intellectual heritage of Ptolemy as well as the Islamic tradition… The torquetum was used for measuring all three types of astronomical coordinates and the conversions between different types of coordinates, which was stipulated by the Ptolemaic planetary theory" ([1029], page 17). The instrument shown in fig. 1.22 belonged to Petrus Apianus (1497-1552). We are therefore told that the mediaeval Turks "revived" the Ptolemaic theory of measurements, manufacturing the necessary tools after many years of oblivion – namely, fifteen hundred years later than the "ancient" Ptolemy.

As we are beginning to realise, the mediaeval Ottoman turketum was a contemporary of the Ptolemaic devices. These instruments were made in the XV-XVII century.

Fig. 1.21. The astrolabe of Georg Hartmann from Nuremberg. We see both the front and the reverse sides of the instrument. Taken from [1029], page 15.

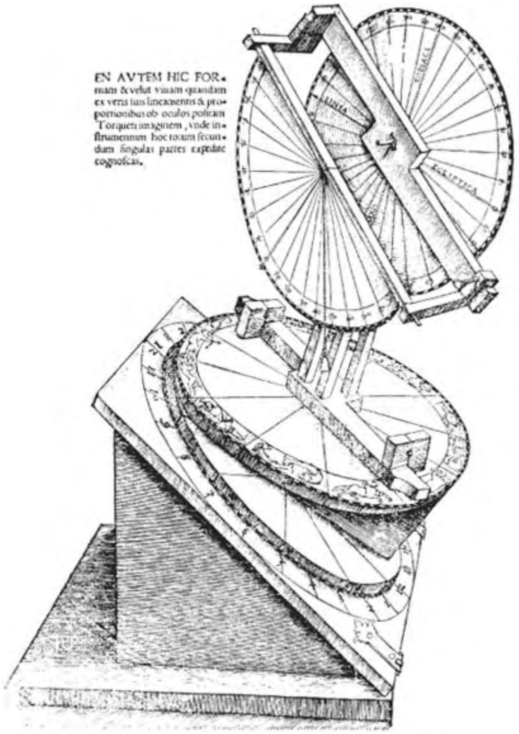

Fig. 1.22. A mediaeval instrument known as turketum ("Turkish"). Designed for estimating several types of celestial objects' coordinates. It was also utilised in Ptolemaic planetary theory (Werner, 1533). Taken from [1029], page 18.

7. TIMEKEEPING AND TIMEKEEPING DEVICES IN MEDIAEVAL ASTRONOMICAL OBSERVATIONS

As we have pointed out earlier, in order to conduct precise astronomical observations, the ancient astronomers needed a chronometer with a minute hand or some equivalent thereof. It would be expedient to recollect the history of mediaeval timekeeping in this respect in order to compare the precision of mediaeval timekeeping devices to the relative precision of the coordinates included in mediaeval star catalogues, the Almagest catalogue in particular.

In general, it has to be mentioned that the very concept of time was rather idiosyncratic in the Middle Ages. The analysis of the ancient documents demonstrates that this concept differed from the modern to a great extent. In particular, time was often considered "anthropomorphic" before the invention of the clock – more specifically, its character and speed would depend on the nature of events. As we already reported in Chron1, "before the XIII-XIV century timekeeping devices were a rarity and a luxury. Sometimes even the scientists would lack them. The Englishman Valcherius … regretted the fact that the precision of his lunar eclipse observations of 1091 was impaired by the absence of a chronometer" ([1461], page 68). Timekeeping devices of low precision were introduced in the Middle Ages: "the usual timekeeping devices in mediaeval Europe were sundials … hourglasses and clepsydrae. However, sundials were only useful for sunny days, and clepsydrae remained a rarity" ([217], page 94).

In fig. 1.23 we see the astronomical rings of the XVII-XVIII century, which were used for telling the time by the Sun in particular. The method of their use is shown in an old drawing that we reproduce in fig. 1.24. In fig. 1.25 one sees an old hourglass.

Mass production of clepsydrae falls over the XIII-XIV century. Clepsydrae were used by Tycho Brahe (1546-1601). He used them in order to measure planetary velocities ([954], page 36). In the Middle Ages "the clepsydra was a popular device, its low precision notwithstanding. In order to make them more precise, the constructors of the clepsydrae had to take into account the fact that the water doesn't leave the vessel at a constant speed – the latter depends on the pressure, that is to say, the greater the level of water in a vessel, the greater the pressure. The constructors of the clepsydrae improved the construction somewhat, making it more complex, so that the clock wouldn't slow down as the vessel on top emptied… However, clepsydrae had the tolerance of around 10-20 minutes per day, and even the best scientists of the epoch couldn't think of a way to make them substantially more precise" ([288], pages 32-33).

At the end of the IX century candles were used widely for timekeeping purposes. For instance, Alfred, King of England, took candles of different length along on his voyages and ordered to light them one after another ([217], page 94). This method of timekeeping was still used in the XIII-XIV century – in the reign of Charles V and other monarchs of the epoch. Timekeeping candles were known as "the fire clock".

Fig. 1.23. An instrument of the XVII-XVIII century that was used for solar timekeeping, among other things. Taken from [1029], page 21.

Many countries preserved this timekeeping method for a long time. "The Japanese, for example, used timekeeping devices consisting of various incense sticks leaning one against another as recently as 200 years ago. One could 'smell' the hour by their aroma, as it were. The Europeans used 'fire clocks' as well – they were candles with special markings" ([954], page 37). We can see that all these "ancient" timekeeping methods were used relatively recently; one must think, they were invented not so very long ago.

"Fire clocks" were used in China for a long time as well. Special kinds of powdered wood were made into a paste, which would then be rolled into sticks of various shapes – spirals and so on. Occasionally, metal balls were tied to these sticks in certain places. As the stick burned, they would fall into a vase and make a sound. "The precision of 'fire clocks' also left much to be desired – apart from the difficulty of making perfectly uniform sticks and candles, the speed of their combustion always depended on the atmospheric conditions (wind, fresh air supply etc)" ([288], pages 30-31).

The hourglass was another popular timekeeping device of the Middle Ages. "The precision of the hourglass depends on the stability of the sand flow. In order to make the hourglass more precise, one needs to use sand of as uniform a texture as possible, soft, dry and forming no lumps inside the vessel. Mediaeval craftsmen of the XIII achieved this by boiling the mixture of sand and marble dust with wine and lemon juice, skimming it, then drying and repeating the process nine times over. All of these measures notwithstanding, the hourglass remained a timekeeping instrument of low precision" ([288], page 30). In the XII century, the secular rulers of Mons who wanted to begin a process at a given time had to consult with the ecclesiastic authorities about the time of day" ([1037], pages 117-118).

Nowadays it is believed that the first mention of a mechanical chronometer dates from the end of the VI century A.D. ([797]). Then the devices disappear for a long time to resurface already during the Renaissance. According to the specialists in the history of sciences, "the first mechanical clock was made by the ingenuous and curious Italian craftsmen in the XIII century" ([954], page 38). The principle of their construction is simple enough – a rope with a weight on its end is woven onto a horizontal shaft. The weight pulls the unwinding rope, which rotates the shaft. If we are to attach a hand to the shaft, it will tell the time. Despite the simplicity of the principle, its practical realisation required a stable slow rate of shaft rotation. This purpose was achieved by means of using numerous wheels, which transferred the rotation of the shaft to the hand, and clever regulators of all kinds, installed to make the shaft rotation rate more or less uniform. "Mechanical clocks were constructions of formidable size. Enormous clockwork mechanisms were installed on the towers of cathedrals and palaces" ([954], page 38). A flywheel from Tycho Brahe's clock had 1200 notches and a diameter of 91 centimetres" ([288], page 35). "The wheels of some clocks weighed hundreds of kilos. Due to the large weight of their parts and substantial friction, wheel-based mechanical clocks required lubrication and constant maintenance. The daily tolerance rate of such clocks equalled several minutes" ([288], page 35).

"It was only in the XV century that the spring replaced the shaft and rope in clockwork mechanisms.

The weight of clocks was reduced dramatically. Craftsmen of the early XVI century mastered the construction of mobile spring-based clocks that weighed 3 or 4 kilos. They were the rather heavy ancestor of the modern mechanical watch" ([954], page 39).

The invention of the clock with a minute hand must have been followed by the compilation of a more or less precise longitudinal star catalogue. What is the significance of the minute hand? The matter is that the celestial sphere and all the objects seen upon it rotates at the speed of one degree per 4 minutes; therefore, a star passes 15 arc minutes per minute of time. Star catalogues contain coordinates of stars indicated with arc minutes – therefore, in order to make the catalogue precision tolerance equal circa 15 arc minutes, one needs to be able to track the time interval of one minute on a timekeeping device. The tolerance of circa 10 minutes (as in the Almagest, for instance) requires the ability of measuring 40-second intervals reliably. Higher precision of a catalogue requires a higher precision of timekeeping devices. Of course, the observers could use their intuition for the measurement of short time intervals (one minute and less), but this would introduce subjective errata into the catalogue.

Thus, the ancient astronomers who claimed their catalogues to have a tolerance of 10' needed to have a chronometer with a minute hand or some analogue thereof at their disposal. However, Ptolemy, who gives us a detailed description of all the instruments required for the measurements of stellar coordinates (the armillary sphere etc) doesn't mention any chronometers and altogether refrains from the discussion of the timekeeping problem and its direct relation to the observations of the celestial sphere, which is in a constant motion.

The hypothesis that chronometers with a minute hand could exist in the II century A.D. contradicts Scaligerian information about the history of timekeeping devices, as we shall shortly see.

Also, the above implies that if we really discover some sort of catalogue whose precision tolerance equals 10 arc minutes as declared by the author of the Almagest, and this tolerance is verified by statistical research, we shall have a good reason to assume that the compiler of the catalogue was using a clock with a minute hand or some equivalent of it.


VSVS ANNVLI
ASTRONOMICI PER
Gemmam Phrysium.

MODIS OMNIBVS ORNATISSIMO
Ac vere Nobili Domino Ioanni Khreutter,
Serenissimę Reginę Hungarię Secretario
Gemma Phrysius S. D.

INter multa variaq; animantium genera, quæ diuersissimis ac admiratione dignis effinxit natura dotibus, vix inuenias vir ornatiss. aliquod, quod minus suo fungatur officio atq; humanum genus. Quod quum a Deo Opt. Max. creatum sit perfectissimum, ratione illa diuina animi parte præditum, qua & ea quæ recte fiunt eligeret, sectareturq;, & ea quæ præter officiũ sunt fugeret detestareturq;, nihil minus agit, imo quasi quadam animi
On

Fig. 1.24. The astronomical rings of Gemma Frisius. "A portable equatorial instrument that could be used at any latitude … for solar timekeeping, as well as many other approximated astronomical observations (Apianus, 1539). Taken from [1029], page 21.

According to the history of timekeeping, the hour hand was introduced into the mechanism of a clepsydra in the XIII century A.D. ([544], Volume 4, page 267) or even later. The timekeeping devices in question had no pendulum, and were therefore of low precision. It was only in the XIV century A.D. that different cities of mediaeval Europe got tower clockwork mechanisms (Milan in 1306 and Padua in 1344). It is reported that they were built by a certain Dondi Horologiu. Clocks with springs moved by a weight were only brought into existence in the XV century. Walther was the first to use them for astro-

Fig. 1.25. Ancient hourglasses. Cambridge, Whipple Museum. Taken from [1029], page 31.

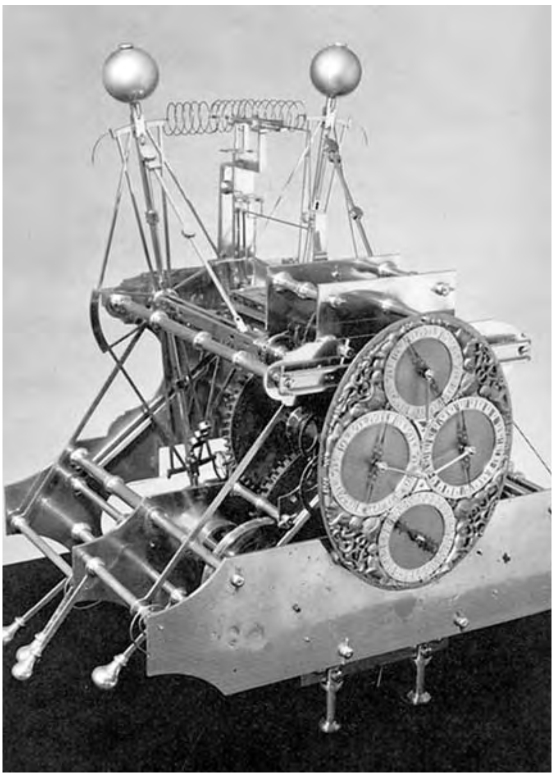

Fig. 1.26. The first chronometer created by John Harrison in 1735. The height of the instrument is 408 millimetres. Taken from [1029], page 140.

nomical observations, followed by many others up to Tycho Brahe ([544], Volume 4, pages 267-268).

According to the history of sciences, "various mechanical clocks only had the hour hand initially. In the middle of the XVI century the minute hand was introduced, and the second hand's invention took place 200 years later" ([954], page 39). The invention of the mechanical clock's minute hand is usually dated to 1550 A.D. ([288], page 36). It is believed that the first chronometer was only constructed in the XVIII (1785, by John Harrison). Harrison lived around 1683-1776 ([1029], page 139). Harrison's chronometer is a complex enough instrument; it can be seen in fig. 1.26.

The modern mechanical clock, including the pendulum, was invented by Huygens in 1657 ([797]). In 1561 the Kassel observatory was built – a unique construction, since it was the first to embody the principle of rotating roof (a device used in most modern observatories). After the death of Regiomontan and Walther, Landgrave Wilhelm IV of Hessen-Kassel (1532-1592), the creator of said observatory, conducted extensive observations of immobile stars (see Chapter 11 below). In general, "the primary purpose of the Kassel observatory was the compilation of a star catalogue ... The most remarkable innovation was the clock used for timekeeping and measurements involving the motion of the celestial sphere. The construction of a clock whose precision was adequate for this purpose owes its successful implementation to the mechanical genius of Bürgi [1522-1632 – Auth.], and, in particular, to his discovery that the clock can be regulated by the pendulum – apparently, he hadn't made any attempts of making this invention public, and so the pendulum was reinvented before it could be acknowledged by everyone [in re the discovery of Galileo and Huygens – Auth.]. By 1586, the positions of 121 stars were registered with the greatest care, but the complete catalogue, which was supposed to contain over 1000 stars, has never been finished" ([65], page 118).

The activity of Tycho Brahe, who worked in the same epoch, soon completely outshone the efforts of the Kassel observatory. It is curious enough that the scientists of the Kassel observatory already used refraction compensation to counteract the errata introduced by the refraction of sunlight in the atmosphere ([65], page 118).

It was only in the time of Huygens that the clock became an integral part of many astronomical instruments: "One of the inventions made by Huygens completely revolutionized the art of precise astronomical observation. Huygens attached the pendulum to the clock that was set in motion by weights, in such a manner that the clock maintained the pendulum in motion, which, in turn, regulated the motion of the clockwork.

It is likely that Galileo planned to unite the pendulum and the clockwork mechanism towards the end of his life, but we have no proof that he ever managed to make this idea come alive.

This invention has given us the opportunity to make precise observations, and, noting the gap between two stars crossing the meridian, deduce their angle distance to the west or the east, knowing the speed of the celestial sphere's motion.

Picard was the first to appreciate the importance of this invention for astronomy, introducing correct timekeeping in the newly built Paris Observatory" ([65], page 177).