Unsuccessful attempts of dating the Almagest. Reasons for failure. Our new approach and a brief account of our results

1. THE ATTEMPT TO DATE THE ALMAGEST BY A COMPARISON TO THE CALCULATED CATALOGUES REFLECTING THE MOTION OF THE FASTEST STARS

1.1. The comparison of the Almagest catalogue to the calculated catalogues

In Chapter 1 we refer to the algorithm of recalculating the modern positions of celestial objects backwards "into the past". Thus, what we have at our disposal presently is the Almagest catalogue compiled in ecliptic coordinates in some unknown epoch t_A, and the set $\{K(t)\}$ of the calculated star catalogues. They reflect the real situation on the celestial sphere that we computed for a given time moment t. Let us try and determine the desired value of the date t_A, or the epoch when the Almagest catalogue was compiled. We shall begin with the following idea which appears quite simple and try to compare the positions of individual stars in the Almagest to their positions in the calculated catalogues $K(t)$; after that we shall try to select such a value t^* for the evaluation of the date t_A that it would make the Almagest data correspond to those contained in the catalogue $K(t^*)$ in the best way possible.

We shall refrain from going into detail about the quality criteria of such correspondence and merely define the meaning of "comparing the Almagest to catalogue $K(t)$ with a given t value". What this implies is selecting the same coordinates from catalogue $K(t)$ and the Almagest. The comparison in question makes year t serve for the alleged dating of the observations that the Almagest catalogue is based upon. Therefore, in order to compare the coordinates of the stars in the Almagest with their coordinates in the calculated catalogue, one has to set the Almagest ecliptic into the same plane as the ecliptic of the calculated catalogue $K(t)$.

However, such a superimposition shall allow for nothing but latitudinal comparison, whereas we also need to compare stellar longitudes. In other words, we shall have to impose the Almagest star atlas over the real one for epoch t, supposing t to be the real time when the Almagest author performed his observations. This requires marking the vernal equinox point for epoch t on the Almagest ecliptic. This point is to be selected in such a way that the average longitude error for the Zodiacal stars of the Almagest would equal zero. Bear in mind that we are using the table of traditional identifications of the Almagest stars with the modern star chart as given in [1339] for our comparison with the longitude of the relevant stars from the catalogue $K(t)$. It isn't that formidable a task to select such an equinox point. As it is known (qv in [1040] and [1339]) that $t = 18.4$, or corresponds to the Aries arc sign on the Almagest ecliptic for 60 A.D., shifting with the speed of roughly 49.8" for each year t – the precession speed, that is.

We cannot quite evade errors in our choice of the vernal equinox point on the Almagest ecliptic with the method indicated above, which is optimal statistically. Its complete evasion would be achieved if we merely compared stellar latitudes without taking the longitudes into account whatsoever. This is what we shall do below, in Chapters 3-5. We shall analyze the latitudes and the longitudes separately. The considerations given in the current section are of a preliminary character.

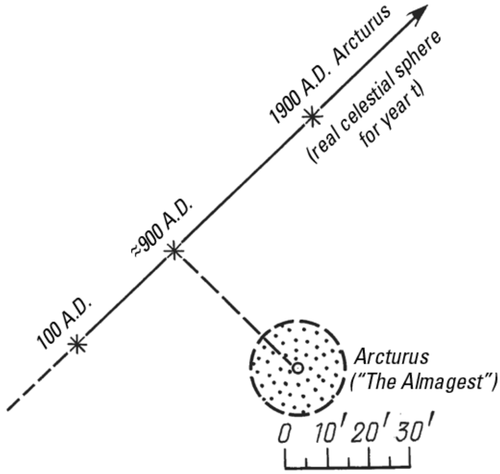


Fig. 3.1. The motion of the real Arcturus as compared to its position specified in the Almagest. This graph doesn't account for the systematic error made by Ptolemy or compensate it.

1.2. The attempt of dating the Almagest catalogue by proper movements of individual stars

Let us choose nine of the fastest stars for comparison, indicated in the Almagest according to [1339]. These are the stars, whose proper movement speed exceeds 1" per year. Their list is as follows:

α Cent (969) – 4.08" per year,
o^2 Eri (779) – 3.68" per year,
α Boo (110) = Arcturus – 2.28" per year,
τ Cet (732) – 1.92" per year,
α CMa (818) = Sirius – 1.33" per year,
γ Ser (265) – 1.32" per year
ι Per (196) – 1.27" per year,
α CMi (848) = Procyon – 1.25" per year,
η Cas (180) – 1.22" per year.

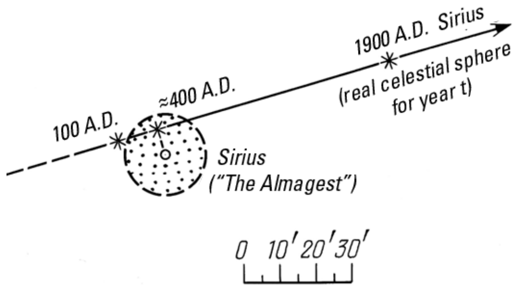


Fig. 3.2. The motion of the real Sirius as compared to its position specified in the Almagest. This graph doesn't account for the systematic error made by Ptolemy or compensate it.

All these stars are contained in the Almagest, according to traditional identifications ([1339]). The numbers given to them by Bailey in the serial numeration of the Almagest are in parentheses. Let us represent each of these Almagest stars as a circle without any shading, see figs. 3.1-3.8. We decided to omit α Centauri, since the coordinates of this star which lays far to the south are given in the Almagest with the gigantic 8-degree error. In fig. 3.4, apart from the Almagest star 779, one can also see the neighbouring stars 778 and 780 and the trajectories of real stars

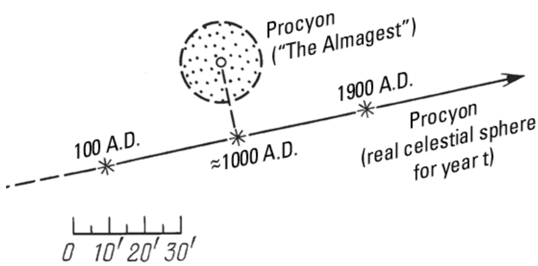


Fig. 3.3. The motion of the real Procyon as compared to its position specified in the Almagest. This graph doesn't account for the systematic error made by Ptolemy or compensate it.

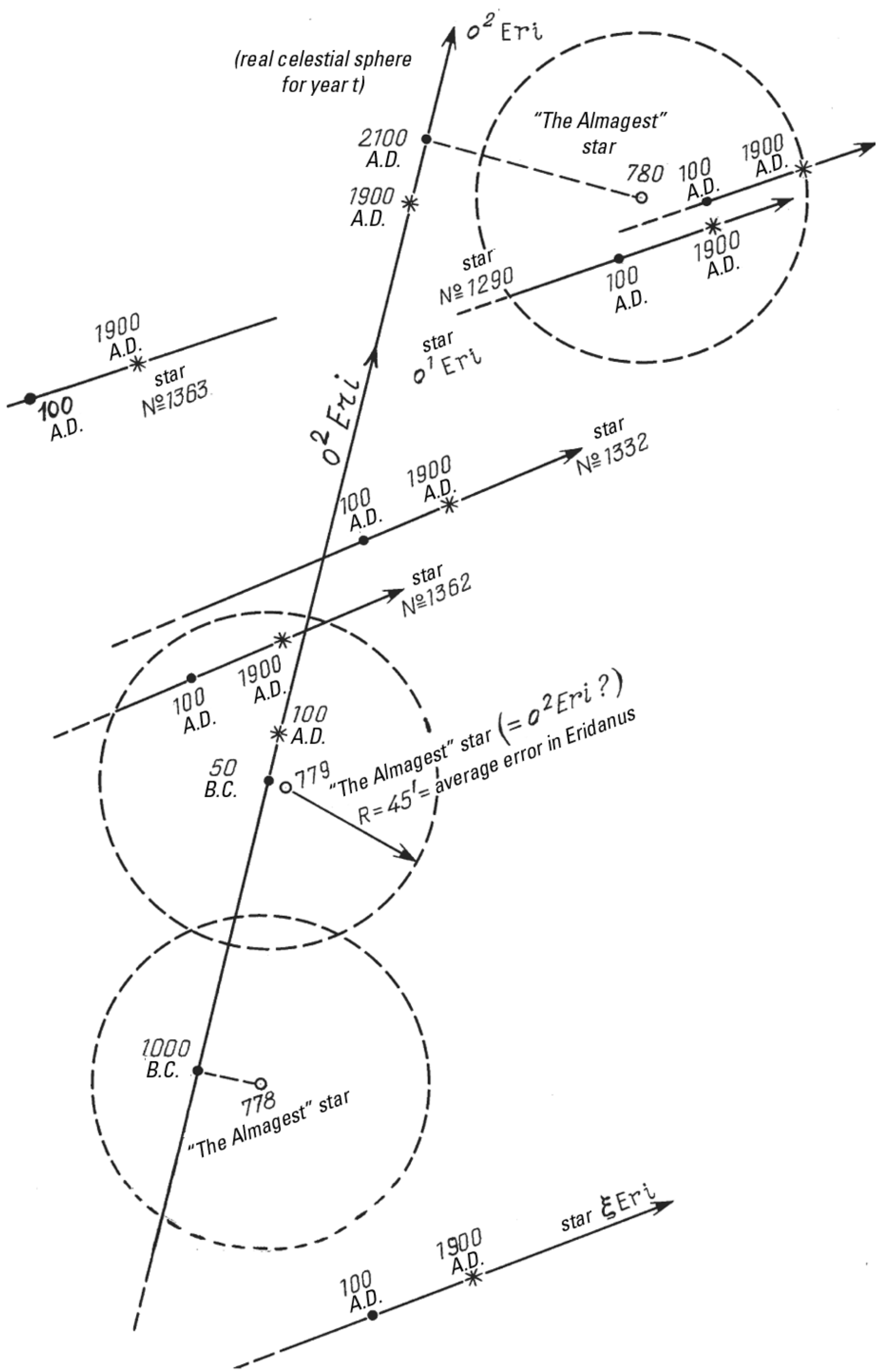


Fig. 3.4. The motion of the real stars o² Eri and ξ Eri as compared to the Almagest data. This graph doesn't account for the systematic error made by Ptolemy or compensate it. The numbers of the stars are given in accordance to a modern catalogue ([1197]).

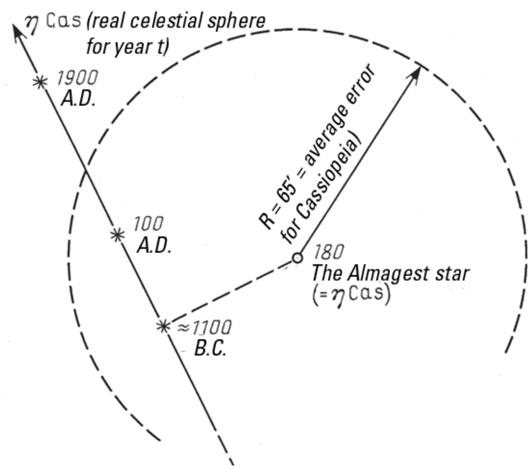


Fig. 3.5. The motion of the real star η Cas as compared to its position specified in the Almagest. This graph doesn't account for the systematic error made by Ptolemy or compensate it.

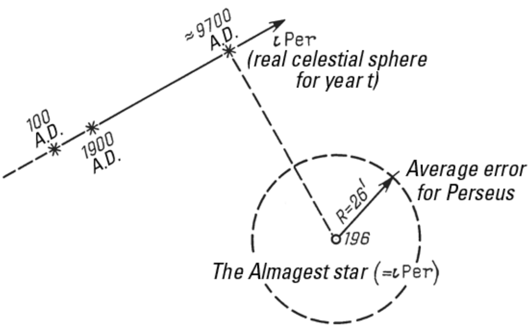


Fig. 3.6. The motion of the real star ι Per as compared to its position specified in the Almagest. This graph doesn't account for the systematic error made by Ptolemy or compensate it.

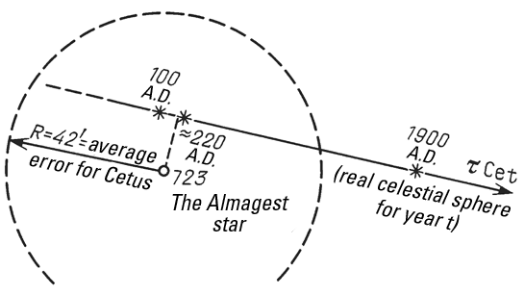


Fig. 3.7. The motion of the real star τ Cet as compared to its position specified in the Almagest. This graph doesn't account for the systematic error made by Ptolemy or compensate it.

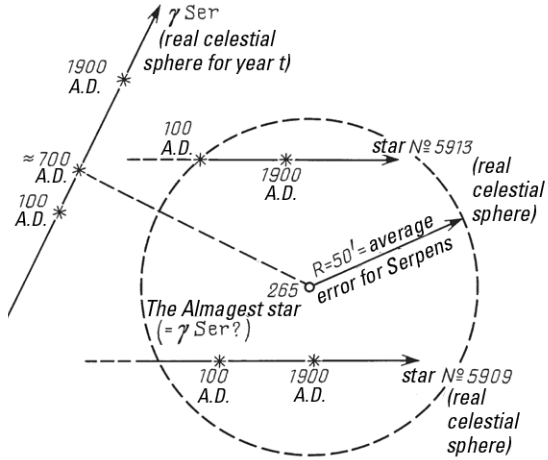


Fig. 3.8. The motion of the real star γ Ser as compared to its position specified in the Almagest. This graph doesn't account for the systematic error made by Ptolemy or compensate it. Star numbers are given according to a modern catalogue ([1197]).

numbered 1332, 1362 and 1363 from the catalogue ([1197]). Thus, we have eight stars left.

Let us now regard the small neighbouring areas of each of these eight stars in Ptolemy's star atlas. We shall be using these star coordinates as given in the Almagest. Each of these areas contains one of the eight fast stars listed above. Furthermore, we share the opinion [1339] that Ptolemy did in fact observe all of these eight stars, and that they are really present in his catalogue.

Now let us superimpose the star atlas compiled from the calculated catalogue $K(t)$ which reflects the state of the real celestial sphere for epoch t, over Ptolemy's star atlas compiled from the Almagest; we shall be using the method described above, and perform this procedure for every t moment. We shall now draw our eight fast stars among the stars of the Almagest.

The method of imposing the calculated atlas $K(t)$ over Ptolemy's atlas depends on the choice of epoch t. Moreover, each of the eight fast stars changes its position in relation to the other stars from the calculated catalogue $K(t)$ with an alteration of t. Thus, the way these stars shall be represented on Ptolemy's atlas shall also depend on the time t. We will come up with eight new trajectories on Ptolemy's atlas corresponding to the shift of our eight fast stars after the alteration of t. These trajectories can be seen in figs. 3.1-3.8. Let us emphasize that we are not yet taking into account the systematic error in stellar locations that we discovered the Almagest's compiler to have made. We shall relate the story of this error in detail below.

What are the t moments that we are considering now when the real fast stars are the closest to how they were represented on Ptolemy's atlas?

Generally speaking, these moments vary from star to star. For the eight stars listed above we shall mark them as $t_1, t_2, \ldots, t_8$. If it turns out that all the values of t_i $(1 \leq i \leq 8)$, or a considerable part of them at the very least, turn out to be close to each other as well as some averaged value of t^*, it shall be strong argumentation in favour of the theory that the true time of the Almagest's author's observations is close to t^*.

However, this doesn't appear to happen. Indeed, the values t_i are chaotically scattered across the time interval $-70 \leq t \leq 30$), or 1000 B.C. – 9000 A.D.! The range is just too great. Let us compile the results into table 3.1 to make them more illustrative. The fact that the individual datings t_i are spread across this great a range is hardly surprising. The matter is that each of the eight stars under comparison is represented in the Almagest with a certain error which is rather serious.

The idea of the possible rate of this error for an individual star can be obtained from the average arc declination in the constellation that the star in question is part of. Under the arc declination we understand the gap between the star's position in the Almagest and its true calculated position. Strictly speaking, the indicated average error depends on the alleged dating of the Almagest – due to the proper movements of stars, for instance. However, the stars on the celestial sphere are almost immobile for the most part. It appears that the rate of this average error is only marginally dependent on the epoch that the stellar coordinates are calculated for. The precision level that is of interest to us allows to disregard this dependency.

In order to calculate the average error rate, we have used the comparison table that contains the star positions in the Almagest together with their real positions for 130 B.C. that we encounter in the work of Peters and Knobel ([1339]) – calculated for the epoch of the "ancient" Hipparchus, that is. Let us draw the "precision circle" around the point that represents a fast star in the Almagest whose radius will equal the average error rate for the constellation that contains the star in question, qv in figs. 3.4-3.8. The projection of this circle over the trajectory of the calculated star that reflects the movement of a real fast star across the celestial sphere shall give us an idea of the possible error rate pertinent to the individual dating t_i by the star in question as compared to the real date of the catalogue's compilation. Let us also point out that the individual star measurement errors that we know nothing about can differ from the average error rate drastically. The radius of the "precision circle" for Arcturus, Procyon, Sirius and other named stars was chosen as equalling 10', or the Almagest catalogue scale grading value. See figs. 3.1-3.3.

Table 3.1. Approximate datings of the Almagest catalogue by the proper movements of eight fastest stars observable with the naked eye.

Star name	*Dating closest to the star observation time in the Almagest*	*Minimal distance to the star of the Almagest*
Arcturus = α Boo	900 A.D.	40'
Sirius = α CMa	400 A.D.	10'
Procyon = α CMi	1000 A.D.	20'
o^2 Eri	50 A.D.	5'
η Cas	1100 B.C.	40'
ι Per	9700 A.D.	70'
τ Cet	220 A.D.	15'
γ Ser	700 A.D.	80'

1.3. Why the dating of the Almagest by individual star movements gives us no reliable result

The question that inevitably arises in this regard is whether the results achieved with the use of one or several of the eight stars listed above can be trusted more? In that case, this is the star which we must use for the purpose of evaluating and dating Ptolemy's research, rejecting the datings based on all the other stars as not reliable enough. It is natural to use the stars whose coordinates are the most correct in the Almagest. But how does one choose them?

In some works it was suggested to evaluate the precision of Ptolemy's measurements for each of the stars in question basing our judgement on the calculated arc discrepancy for a given star – using the last column of the cited table, in other words. The implication would be that the coordinates of the star o^2 Eri were measured by Ptolemy with the precision rate of 5', for instance, and those of Arcturus – with the precision rate of 40'. This is exactly what the authors of [273] Y. N. Yefremov and Y. D. Pavlovskaya had done. They had tried to date the Almagest by proper movements and worked with the same list of 9 stars in particular. This approach would yield a dating which would be close to the Scaligerian – 50 B.C., qv in table 3.1. The evaluation of the possibility that this dating is erroneous is a separate issue which we shall consider below. To jump ahead very briefly, we shall merely state that the possible error rate of Yefremov and Pavlovskaya's method was estimated perfectly unrealistically in [273].

This approach instantly leads us to the following set of questions. The first one concerns the rather absurd situation in which all three stars of the first magnitude out of nine, namely, Arcturus, Sirius and Procyon (and ones that have names of their own in the catalogue at that) were measured by Ptolemy very roughly, with error rates approximating an entire degree. Yet the dim and poorly-visible star o^2 Eri was for some reason measured with the utmost precision, the discrepancy equalling a mere 5'! Let us explain that the magnitude of this star according to modern measurements equals a mere 4.5, which means it is very dim.

All of this is most bizarre indeed. Such bright and famous stars as Arcturus, Procyon, Regulus and Spica must have served Ptolemy in his research as control points, or, at the very least, their coordinates were measured with the utmost care and precision. Their exceptional importance to ancient astronomy is reflected in the very fact that they have own names in the Almagest. There are even special sections of the Almagest concerned with the measurements of some of them. Therefore the precision of their coordinate calculation must have been very high indeed (see [968], for instance). At the same time, there is nothing very noticeable about the star o^2 Eri. It cannot be distinguished from the stars surrounding it, them being just as dim.

Furthermore, the star traditionally associated with o^2 Eri is merely described as an "average star" in the Almagest. Therefore, we would be justified to ask another perplexed question after taking a look at fig. 3.4. Why would the Almagest star #779 possibly be identified as o^2 Eri? It is perfectly clear that this is a conclusion one can only arrive at in case when the coordinates of the real star o^2 Eri and the star #779 from the Almagest correlate with each other optimally – better than those of o^2 Eri and the star #778, for instance. However, due to the significant proper motion velocity of o^2 Eri this clearly implies that its identification as any star of the Almagest is greatly dependent on the time we date the Almagest to.

For instance, if we knew that the Almagest was written in 1000 B.C., we could identify o^2 Eri with the Almagest star #778, and then successfully "date" the Almagest to the very same year 1000 B.C. judging by the minimal possible distance between o^2 Eri and the star #778, which would serve as "sound proof" of our a priori dating.

A propos, this identification makes the concurrence between the coordinates of o^2 Eri and the Almagest even better than the traditional version, as one can plainly see in fig. 3.4. If we assume that the Almagest was written in 1500 A.D., or the XVI century, for instance, we might identify the star o^2 Eri as the Almagest star #780 and date it to the late Middle Ages, or even a "future epoch", qv in fig. 3.4.

It is clear that ruminations of this sort lead to a vicious circle. The dating of the observations based on proper star motion requires a reliable identification of said star as one contained in the Almagest, all of this independently from its presumed dating.

However, even if we are to disregard o^2 Eri, we still cannot use the remaining eight fast stars for a secure dating, even now. The dating dispersion is too great for all the different stars. Even the datings made by the stars of the first magnitude out of the eight stars under study (Arcturus, Procyon and Sirius) are scattered over the 600-year interval between 400 A.D. and 1000 A.D., qv in table 3.1.

Furthermore, one needn't forget that the datings deduced in such a manner (900 A.D. for Arcturus) only represent the moments when the real positions of the stars are the closest to those given in the Almagest catalogue.

One also needs to specify the time intervals surrounding these datings for which the deviation values would fall into a range conforming to precision requirements.

The gravity of the situation is all the greater that if we are to use average values for the evaluation of just how precisely this star or the other was measured in the Almagest, we shall be making a certain error a priori, knowing nothing of the individual errors made in the measurement of the stars in question by Ptolemy.

Let us formulate the corollaries:

1. Before one can use the coordinates of a separate star as given in the Almagest for the purposes of dating, one needs to make sure that identifying the star in question as a star observed upon the modern celestial sphere does not depend on a presumed dating of the Almagest, which would lead us to a vicious circle once again.

2. Even for the fastest of stars, the shifts made due to proper motions are small enough inasmuch as the span of the historical period is concerned (see figs. 3.1-3.8). Therefore, a dating would require a selection of stars whose positions in the Almagest would be measured with enough precision. A star than only shifts by 2" in a year will shift by a mere 3.3' over the period of a century.

Therefore, if we want to use an individual fast star for the dating of the Almagest with the precision range of circa 300 years, we must be certain that the precision of this star's position as given in the Almagest does not exceed the discrepancy rate of 10'. According to the estimations of researchers, the real precision of the Almagest is a lot lower in general ([1339]).

The stars whose coordinate precision discrepancy rate exceeds 20' are all but void of utility for us. The dating interval is 1200 years minimum if we are to use them for dating purposes.

This issue is considered in more detail below (see Chapters 5 and 6).

2. AN ATTEMPT OF DATING THE ALMAGEST CATALOGUE BY THE AGGREGATE OF FAST AND NAMED STARS AS COMPARED TO THE CALCULATED CATALOGUES

2.1. The criteria one is to adhere to in one's choice of the stars for the purpose of dating

In section 1 we demonstrate that the comparison of the Almagest with the calculated catalogues $K(t)$ by the eight of the fastest stars doesn't allow us to indicate a t^* value that makes the Almagest correlate with the catalogue $K(t^*)$ in the best possible manner. For each star the value of $t^* = t_i^*$ is unique and differs from the values of other stars significantly. The scatter range for different stars equals several millennia. Therefore, the approach as described above is too rough, and gives us no substantial result.

However, it might turn out that once we make the sample include a lot more stars than eight, we shall come up with such a set of individual datings $\{t_i^*\}$ whose larger part will fall into a rather short time interval. At the end of the day, even an interval of circa 500 years would suffice; in this case we would be given some sort of opportunity to obtain the information concerning the real date of Ptolemy's research (t_A). Apart from that, making the sample more inclusive might enable us the use of mathematical statistics methods for the estimation of the t_A value.

What other stars should one include in the sample? It is clear that only the fast and relatively well-measured stars fit the purposes of dating. These two criteria – proper motion velocity and the record precision in the Almagest, complement each other in general, since the faster the star, the greater the error we can make for its coordinate in the Almagest without affecting the dating by the star in question.

These considerations lead us to the choice of the following stars for the comparison of the Almagest with the calculated catalogues $K(t)$.

1) The stars which move fast enough. Let us choose 0.5" as the annual speed threshold pertaining to a single equatorial coordinate at least α_{1900} and δ_{1900} for the epoch of 1900 A.D., qv in table 1.1).

2) "Famous" or named stars, or the stars which have old names of their own (see table P1.2 in Annex 1).

Naturally, named stars may have received their names already after the creation of the Almagest, which appears to be true for many stars. However, firstly, the stars' names are unlikely to have been forgotten with age, although they may indeed have altered. In other words, named stars of Ptolemy's epoch remain such until the present day. Secondly, the fact that a given star received a name of its own tells us that it had been charged with a particular significance in old astronomy. It would therefore be self-implied that Ptolemy had paid more attention to named stars than to others, which would be manifest in their more precise measurement especially.

Let us choose the interval of $0 \le t \le 30$ as the a priori time interval for our research (1100 B.C. to 1900 A.D., that is). Bear in mind that the letter t refers to the time counted backwards from 1900 A.D. in centuries.

2.2. The "proximity interval" system as applied to certain fast or named stars

Let us merge the lists of fast and named stars from tables P1.1 and P1.2 (from Annex 1) in order to study them together. We shall choose those stars from the multitude that one finds in the Almagest according to [1339]. The resulting list consists of circa 80 stars. Let us calculate the trajectory of every star from this list in the Almagest coordinate grid as we have done in section 1 for the eight fastest stars.

Be sure to mark that for this purpose we have fixed a certain t value as the presumed dating and calculated the location of each star for the epoch t in the ecliptic coordinates of the epoch. This position can be represented as a point on Ptolemy's star atlas – that is, an atlas built from the Almagest catalogue under the assumption that it was compiled in epoch t. Changing the value of the alleged dating t within the range of the historical interval under study, we are making the star, or point, move along Ptolemy's atlas across the stars of the Almagest. As time t alters, the calculated star i moves across the stars of the Almagest (proper star motion as well as the slight shifts of the ecliptic that take place with the course of time). The distance between the calculated point or star and the Almagest star that this star becomes identified as also changes in its turn. The identifications

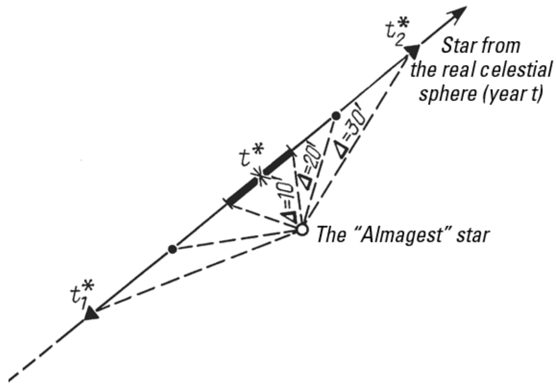


Fig. 3.9. The motion of a real star near the position specified for it in the Almagest.

correspond to [1339]. The distances on the celestial sphere would be measured on the geodetic arc that connects the stars. Bear in mind that geodesic lines on a sphere, or the line of the shortest local lengths, are the arcs of large circumferences or flat cross-sections that go through the centre of the sphere. Such distances on spheres are called arc distances; we shall simply refer to them as "distances".

Let the distance between the stars be minimal for the moment $t^* = t_i$. We have dubbed moment t^* the "individual dating" by a given star in section 1. When t deviates from the t^* value into either direction, the distance between the real calculated star and its representation in the Almagest begins to grow.

Let us conside the dating interval $[t^*_1, t^*_2] = [t_{i1}, t_{i2}]$ where the distance in question does not exceed 30' correspond to every star with the number i from the list. This interval can actually be empty, which shall be the case if the distance between the calculated star and the respective star from the Almagest exceeds 30' for moment t. The centre of the interval shall be defined by value t^*. See fig. 3.9.

The 30' limit for the arc distance between the Almagest star and the corresponding calculated star was chosen with the goal of having most of the Almagest stars stay within it. Indeed, if we are to consider the average square error rate in the arc distance for the Almagest stars to exceed 40' (which concurs with the research conducted in [1339] and [614]), more than half the stars in the Almagest must be represented with the precision rate of circa 30'. We are basing this on the hypotheses of normal error distribution and of error independence as taken for individual stars. Due to the approximate nature of our narrative, possible discrepancies that these presumptions might lead to do not affect our corollaries.

The set of the intervals that we calculate in this manner, or the "proximity intervals", can be seen in fig. 3.10. What we see here is the time axis beginning with $t = 0$, or 1900 A.D., and ending with $t = 30$, or 1100 B.C. Each interval has a centre defined by the optimal dating t_i for a given star. We also mark the points for which the distance between the "Almagest star", or the position given in the Almagest, and the calculated star, equals 10' and 20' (see fig. 3.9). Lines representing distances under 10' are heavier as seen in fig. 3.10. The ends of the intervals are marked with pointers where they stay within the graph.

Many of the stars in our list of fast and named stars do not have a corresponding interval in fig. 3.10. This should imply the interval in question to be:

1) Altogether nonexistent (in cases when the distance between the Almagest star and the calculated star remains greater than 30' in all cases).

2) Failing to cross the a priori interval $0 \leq t \leq 30$ and located beyond the area of the graph.

3) Covering the a priori interval completely.

In the latter case, the coordinates of the star must have been measured with enough precision for the 30' interval; however, one cannot date the observations in the interval between 1100 B.C. and 1900 A.D. by the positions of such stars since their movement is too slow.

Let us give Bailey's numbers of the Almagest stars for which the 30-minute proximity intervals cover the entire interval $0 \leq t \leq 30$ given a priory (see [1339] and [1024]). These are the stars with numbers 35, 36, 163, 197, 222, 316, 318, 375 and 768.

Only partial intervals are given for many stars. This happens when part of the interval is located outside the a priori interval of $0 \leq t \leq 30$ and thus fails to be represented in fig. 3.10.

Next to each interval one sees the number of the corresponding Almagest star in Bailey's numeration. The name of the modern star identified as the current Almagest star, as well as its own special name, in case of its existence, is given next to the equal sign.

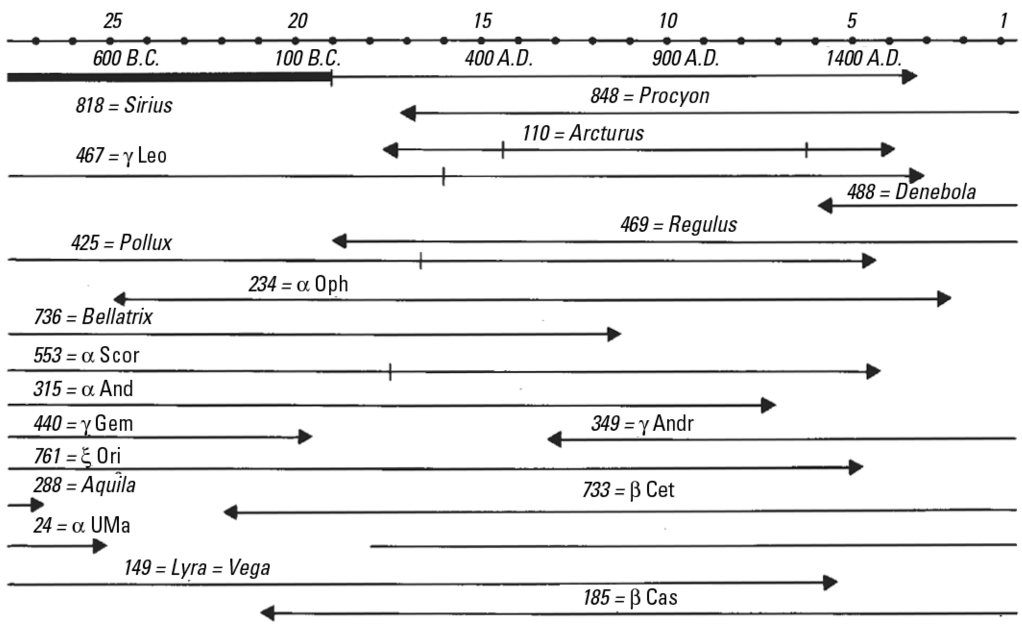


Fig. 3.10. Intervals of maximum proximity between visibly mobile fast or named stars with their corresponding positions as specified in the Almagest.

In fig. 3.12 we reproduce a similar graph for latitudes; the moment $t = 18$ is represented with a dotted line and stands for the Scaligerian dating of the Almagest (around 100 A.D.).

2.3. Dating the Almagest with the suggested method utilizing arc distances of individual stars is an impossibility

Fig. 3.10 tells us very explicitly that time values t which would belong to all the "maximal proximity" intervals simultaneously do not exist. Let us raise the precision threshold starting with the 30' value as chosen above, in order to obtain the desired values of t. The intervals as seen in fig. 3.10 shall grow respectively, with pointers indicating the direction of growth. At some moment, all the intervals shall begin to intersect. Let us see what value of t and precision threshold value it should take for this intersection to occur the first time. It turns out that it takes place with $t \approx 12$, or around 700 A.D., with the precision threshold of about 60', or one degree. If we keep raising the precision threshold, the intersection interval will grow in both directions from the point $t = 12$.

However, we cannot regard point $t = 12$, or 700 A.D., as a reliable enough estimate of the date when the author of the Almagest catalogue carried out his observations since the intersection of all "maximal proximity" intervals in fig. 3.10 only takes place at the precision threshold of 1 degree, which implies the existence of very poorly-measured Almagest stars in this set. The error in the estimate of their position contained in the Almagest equals one degree at the very least.

Furthermore, if we are to estimate the precision of stellar coordinates from below with the aid of the selective average square arc error in the optimal point $t = 12$, we shall have to raise the acceptable error rate value (or the precision threshold) excessively (over 2 degrees). However, such a value of the precision threshold shall make the acceptable "maximal proximity" interval intersection cover the entire period between 500 B.C. and the present (see fig. 3.10). Such a corollary is of zero scientific interest, since it is perfectly understandable that the Almagest was created somewhere in this great time period.

Moreover, the very dating of 700 A.D. is rather un-

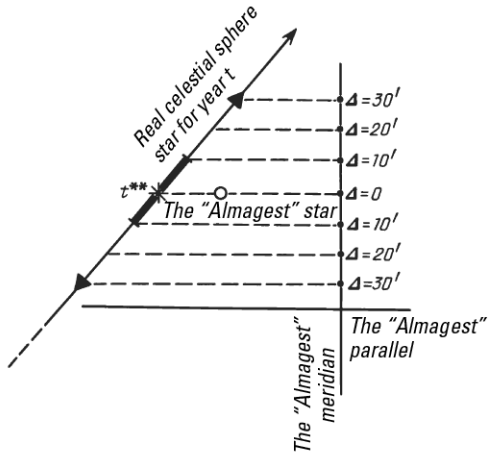


Fig. 3.11. Latitudinal discrepancy for the real calculated star and its position as specified in the Almagest.

stable in the following sense. An alteration in the compound of the stars under study (which is obviously chosen rather arbitrarily) can shift the dating moment rather significantly.

It is clear that such a situation makes all claims of a reliable deduction of the Almagest catalogue compilation date quite void.

2.4. Dating the Almagest catalogue with the suggested method based on latitudinal discrepancies of individual stars also proves impossible

Let us consider another method of calculating maximal proximity intervals for the Almagest stars from our list of fast and named stars. This method is similar to the one described above, the difference being that this time the distance between the Almagest star and the corresponding calculated star is composed of the latitudinal discrepancy and not arc segments. By latitudinal discrepancy we mean the projection length of the interval that connects these two stars over the Almagest coordinate grid meridian (see fig. 3.11). The choice of a latitudinal discrepancy (as opposed to longitudinal, for instance) was made out of the following considerations: firstly, it is well known that the Almagest star latitudes are more precise than the longitudes (qv in [1339], for instance, as well as Chapter 2 of the present book). Secondly, the latitudinal discrepancy does not depend on how we position the Almagest in relation to the calculated catalogue $K(t)$ in terms of longitudes, qv in Chapter 1. Thus, we shall manage to evade making additional errors which may result from such juxtaposition as well as the possible arbitrary choice of the initial longitudinal reference point (see Chapter 1).

In fig. 3.12 we see the resulting maximal proximity interval set for the case when the latitudinal discrepancy represents the distance. Once again, the proximity intervals which cover the entire interval of $0 \leq t \leq 30$, or 1100 B.C. to 1900 A.D., are absent from the graph. The Almagest numbers of the stars whose 30-minute latitudinal proximity intervals cover the interval $0 \leq t \leq 30$ completely are as follows: 1, 35, 36, 78, 111, 149, 163, 189, 222, 234, 287, 288, 315, 316, 318, 349, 375, 393, 410, 411, 424, 467, 469, 510, 713, 733, 760, 761, 768, 812 and 818.

A comparison of fig. 3.12 and fig. 3.10 demonstrates that the longitudes of the Almagest stars under study are indeed a lot more precise than their positions on the celestial sphere defined by both latitude and longitude. This is exactly why one sees more intervals in fig. 3.12 than in fig. 3.10, which represent a greater amount of stars.

Maximal proximity intervals for all the stars in fig. 3.12 apart from two stars in Centaur (935 = 2*g* Cent and 940 = 5θ Cent) also begin to intersect at the level of $t = 12$, or approximately 700 A.D., latitudinal precision threshold equalling 40'. This is somewhat better than the 60' value that we got in the previous case, but still nowhere near precise enough. We are brought to the dating of roughly 700 A.D. once again, but, as in the above case, we cannot consider this result reliable due to the considerations related above; therefore, this method of dating the catalogue gives us no tangible results.

In general, regardless of the fact that the transition from the arc discrepancy to the latitudinal discrepancy helps us rectify the errors of the Almagest to some extent and therefore allows for more precise statistical corollaries, the resulting intervals of possible datings remain too great. They cover the entire period of $4 \leq t \leq 20$, or 100 B.C. – 1500 A.D. Such intervals give us no useful information in re the date of Ptolemy's observations.

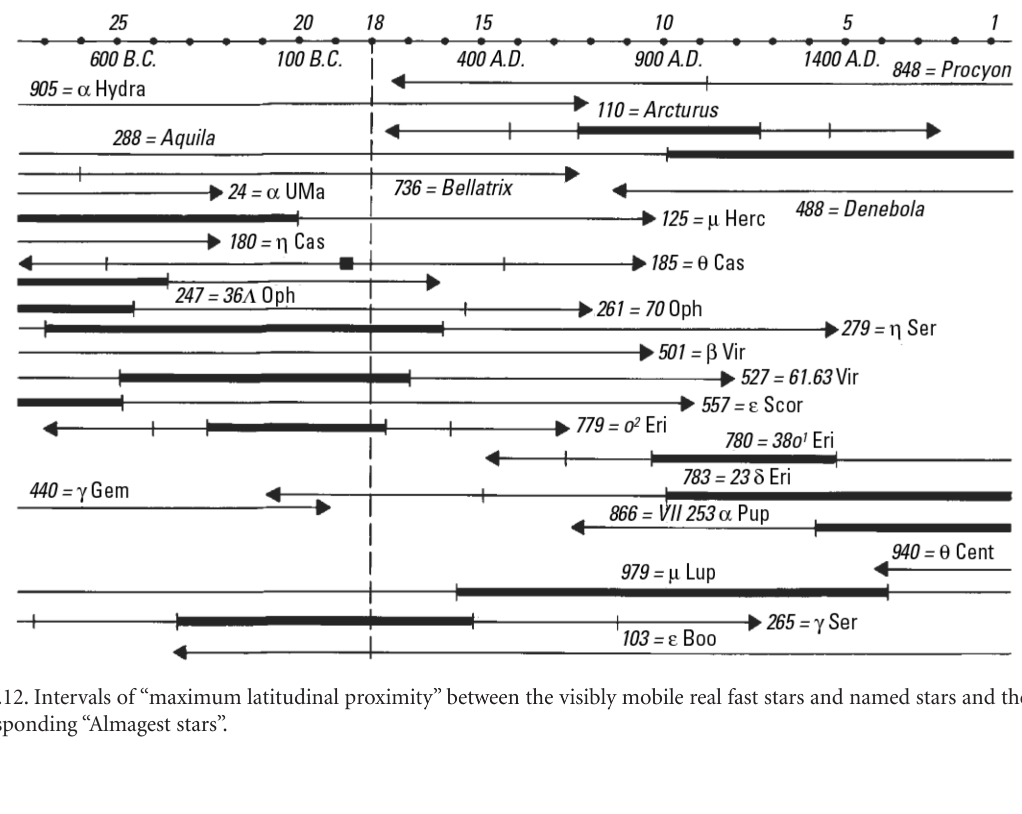


Fig. 3.12. Intervals of "maximum latitudinal proximity" between the visibly mobile real fast stars and named stars and the corresponding "Almagest stars".

3. THE ATTEMPT TO DATE THE ALMAGEST CATALOGUE BY THE MOTION OF INDIVIDUAL STARS AS COMPARED TO THE OBJECTS IN THEIR IMMEDIATE VICINITY

3.1. The varying geometry of stellar configurations as seen against the background of "immobile stars"

In sections 1 and 2 we tried to date the catalogue with rough methods based on various stellar configurations altering over the course of time due to the proper movements of individual stars that comprise them. We have considered each star in the configuration individually, comparing its calculated position to the one given in the Almagest. In order to compare all these positions we had to use the Newcomb theory that describes the movement of the ecliptic coordinate system used in the Almagest across the "sphere of immobile stars" over the course of time.

Let us see what results we can obtain from the method of dating the Almagest that will not use the Newcomb theory. The idea behind a method of this sort is simple. One doesn't compare the positions of individual stars on the "real" theoretically calculated star chart to their positions in the Almagest, but rather the geometry of stellar configurations (which change due to the proper movements of stars) to the configurations from the Almagest catalogue. The only thing required from us for such a comparison is the knowledge of velocity values of the individual stars' proper motion – not the Newcomb theory.

Although the errors resulting from the Newcomb theory are rather small (several orders smaller than the Almagest catalogue grade value), the study of configurations is a lot simpler this way from the calculus point of view.

Proper movements of stars are nowadays measured with great precision with the aid of telescopic observations ([1144] and [1197]). The values of proper star movements and the table that identifies the Almagest stars as their counterparts on the modern star charts comprise the only data that we are to use here. The identification table was borrowed from [1339]; we have omitted the ambiguous cases indicated therein.

3.2. The stars chosen for the experiment

We shall keep comparing the positions of all individual fast-moving stars on the real star chart with their positions as specified in the Almagest. However, now we shall be comparing the positions of the stars on the real chart and in the Almagest to a certain set of referential stars pointed out on the real star chart as well as the Almagest. For this set we have chosen either named stars (Aldebaran, Scheat etc), or those which definitely stand out in brightness amongst the stars that surround them. We excluded the stars whose coordinates might have been affected by refraction from the list of referential stars. 45 stars altogether were chosen, among them such visibly mobile ones as Arcturus, Sirius, Procyon, Capella, Aquila = Altair, Denebola, Caph and Regulus. Thus, the position of a mobile star on the real celestial sphere is determined in reference to a basis that is mobile as well. The resulting picture alters depending on the alleged dating and is compared to the respective picture as reflected in the Almagest.

Let us take the average configuration discrepancy of stellar arc distances as the deviation measure:

$$\overline{\Delta}_i(t) = \frac{1}{N}\sum_{j=1}^{N}\left|\rho_{real}(S_i, O_j, t) - \rho_{Alm}(S_i, O_j)\right|.$$

N stands for the quantity of referential stars, $\rho_{real}(S_i, O_j, t)$ is the arc distance between the star S_i and the referential star O_j on the real celestial sphere of epoch t. Furthermore, $\rho_{Alm}(S_i, O_j)$ is the arc distance between the star S_i and the Almagest star O_j. The time moment t_i when the value of $\overline{\Delta_i}(t)$ reaches its minimum shall be referred to as the individual dating by the star in question. If the individual dating values t_i for all the fast stars of the Almagest catalogue or at least their majority fall into a short enough time interval, said interval should either include the real date of Ptolemy's observations t_A or be located in its immediate vicinity. However, the real status quo appears to be altogether different.

3.3. The behaviour of the individual discrepancies and the average discrepancy

We have studied the behaviour of the $\overline{\Delta_i}(t)$ discrepancies for eight rather fast stars contained in the

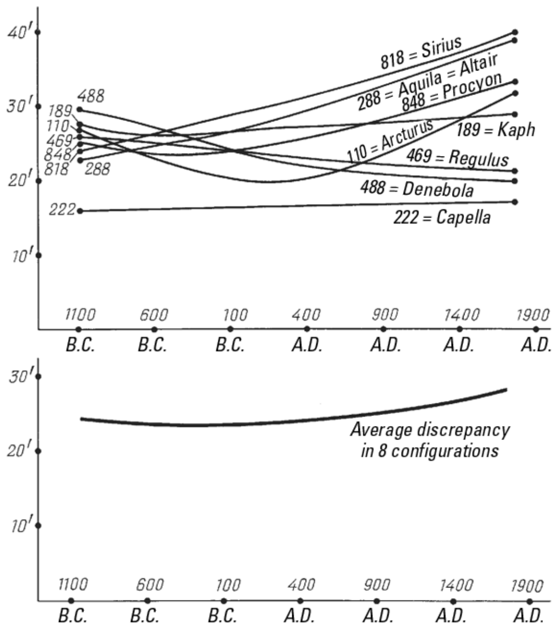


Fig. 3.13. Individual discrepancies for mobile stars and the average discrepancy in eight configurations. It is obvious that one can make no definite conclusions.

Almagest catalogue, namely, Capella (Bailey's number = 222), Arcturus (110), Aquila = Altair (288), Denebola (488), Regulus (469), Sirius (818), Procyon (848) and Caph (189).

We have deliberately chosen the most "famous" and the brightest of the Almagest's fast stars and omitted the dim ones. As we point out above, the coordinates of dimmer stars may be represented in the Almagest very imprecisely. Therefore, their inclusion into the sample can make the scatter range of individual datings a lot wider.

Fig. 3.13 demonstrates the graphs of individual discrepancies for the indicated fast stars $\overline{\Delta_i}(t)$ as t functions as well as the average graph for all these stars. Unfortunately, this graph turns out almost uniform over the entire time interval of 1100 B.C. – 1900 A.D. (see fig. 3.13).

3.4. Negative experiment result

Our refusal to use the Newcomb theory did not lead to the concentration of different datings by individual stars on the time axis. The implication is that the reasons for such a great scatter range of individual datings aren't related to the conversion method as applied to the coordinates of the celestial sphere, but rather relate to the low precision of coordinates offered by the dated catalogue, the possible heterogeneity of the catalogue etc. The latter might be caused by different positions of the instrumental ecliptic during measurements performed in different observatories, which produce different systematic errors for various groups of stars.

In section 5 of the present chapter we shall analyze the coordinates of the Almagest stars as well as the general structure of the Almagest catalogue in order to discover all the factors that might be causing this.

4. THE ANALYSIS OF SEVERAL ERRONEOUS WORKS ON THE SUBJECT OF DATING THE ALMAGEST BY PROPER STAR MOTIONS

4.1. A lot of the errors are not produced by astronomical phenomena and stem from the incorrect application of the methods offered by mathematical statistics

Let us analyze different authors' attempts to date the Almagest by proper star movements.

The articles of the astronomers Y. N. Yefremov and Y. D. Pavlovskaya ([273] and [274]) were published in reference to our publications; they represent an attempt to confirm the Scaligerian dating of the Almagest star catalogue by proper star motion. The corollary formulated in [273] is as follows. The Almagest catalogue can be dated to an early A.D. epoch by proper star motion with the precision threshold of ±100 years. The authors go as far as naming the date of 13 A.D. ±100 years.

In [274], which is a more in-depth publication, the authors formulate their corollary with more caution: "The Almagest star catalogue has thus already been observed in the antiquity; most probably, by Hipparchus. It is however possible that the brighter stars were observed by Ptolemy himself. Some sort of argumentation to support this can be found in the fact that the epochs that we got for Arcturus and Sirius, the two stars of the first magnitude present in our sample, are 2-4 centuries more recent than those for the rest of the stars" ([274], pages 189-190).

However, the actual contents of [273] and [274] imply no such corollary. Let us briefly follow the reasoning patterns of Y. N. Yefremov and Y. D. Pavlovskaya using their more extensive publication ([274]), although everything we say shall also refer to their earlier work ([273]). Let us point out that Y. N. Yefremov hasn't made any scientific publications on this subject ever since the respective publications of said works ([273] and [274]) in 1987 and 1989. However, quite a few of his popular articles have appeared in newspapers and literary magazines. Still it has to be said that both his publications ([273] and [274]) contain errors which were pointed out to their author in our book [METH3]:2, pages 99-103. It would make sense for Mr. Yefremov to correct these errors prior to advertising the results of his research in popular press. Moreover, we are of the opinion that these errors cannot be corrected – in particular, due to the erroneous dating offered by Y. N. Yefremov, qv below.

The dating of star catalogues with the method described in [273] and [274] is based on the comparison of stellar configurations that alter over the course of time with the respective configurations as given in the Almagest. It turns out that the main part in the change of an individual configuration is played by a single star contained therein, the fastest one ("the group of Arcturus", "the group of τ Cet" etc). We shall be using the same terminology.

The dating of a catalogue by an individual configuration is supposed to be such a dating for which the set of pairwise distances between the stars of this changing configuration is the closest to the set of such distances as given in the Almagest. Proximity is defined in the square average sense.

What one gets as a result is naturally a certain approximation of the date when Ptolemy or some other observer who had compiled the Almagest catalogue were making observations – not the actual date. What are the possible discrepancy rates of such approximation, one wonders? There is no factual reply to this question given anywhere in [274].

The discussion of the issue of discrepancy rates for the resultant datings is left out in favour of a reference to the dependency graph of the square average discrepancy between the sets of pairwise distances in the Almagest as well as on the real celestial sphere and the alleged dating of the observations conducted by the author of the Almagest catalogue. We are told that "the epoch T_0 can be estimated with enough confidence, the minimum of the function $\overline{\Delta r^2}(t)$ being drastic and deep" ([274], page 183). However, the illustration that the authors of [274] are referring to (page 185, ill. 3) implies that the alteration of the alleged dating by 1000 years makes the value of the square average discrepancy $\sqrt{\overline{\Delta r^2(t)}}$ alter by a mere maximum of 13' for all configurations except for a single group, that of o^2 Eri. See more about this group below.

Let us see how significant the 13' deviation from the square average discrepancy really is for the situation regarded by Y. N. Yefremov and Y. D. Pavlovskaya. The Almagest scale grade value equals 10', whereas the real precision threshold of the stars in the Almagest estimated as the square average arc discrepancy equals roughly 30' (see [1339] and [614]). If we are to base our estimations on the proper movements of the stars under study, it will imply that the precision estimate according to the method offered in [274], which is based on the minimal square average configuration discrepancy, must allow for the value of this discrepancy to fluctuate within a much greater range than 13' – circa 20'-30'. This leads to the dating intervals of 2-3 millennia. In other words, the possible discrepancy rate for the dates cited in [274] equals 1000-1500 years. See more details concerning the precision of the method related in [273] and [274] below. However, dating the observations performed by the Almagest compiler with such low precision doesn't allow for making a distinction between Ptolemy's epoch and our age, let alone the Scaligerian datings of the respective lifetimes of Hipparchus (II century B.C.) and Ptolemy (II century A.D.). Such a result is of zero scientific value. It is obvious that the Almagest was created during the last two millennia at any rate.

Therefore, this error, as well as the ensuing mistakes made by the authors in question, is of a mathematical nature and not astronomical. The methods of mathematical statistics are either misused or altogether neglected. The claims made by Y. N. Yefremov in re the alleged "high precision" of his methods don't hold up to the simplest criticisms. It is most peculiar that Y. N. Yefremov keeps insisting on the veracity of his erroneous results in the field of Almagest-dating publicly after all these years, the situation being as described above. This concerns his numerous public speeches and popular magazine and newspaper publications oriented at the general public.

4.2. The data in Y. N. Yefremov's works on the dating of the Almagest were tailored to fit the desired result

Y. N. Yefremov and Y. D. Pavlovskaya claim in [274] that the star catalogue dating method that they offer was tested on three veraciously dated catalogues – namely, the catalogues of Ulugbek, Tycho Brahe and Hevelius, and that the application of the method in question to all three catalogue gave an incredibly precise result. The dates when the catalogues of Tycho Brahe were compiled were "restored" with the precision threshold of 30-40 years, and Ulugbek's catalogue, the least precise of the three, was dated with the mind-boggling precision of ±3 years!

However, one cannot overlook the alarming circumstance that each of these datings was calculated by its own stellar configuration – namely, the datings for the catalogues of Tycho Brahe and Hevelius were obtained from the Arcturus groups, and the dating of Ulugbek's catalogue comes from the data obtained from the group of τ Cet. Other stellar configurations for each of the three catalogues in question aren't considered at all. Why would that be? We shall promptly answer this question.

Furthermore, the main result of Y. N. Yefremov and Y. D. Pavlovskaya concerning the dating of the Almagest is also de facto obtained from a single solitary configuration – group o^2 Eri, although they make formal references to having studied 13 configurations. The analysis of the datings that they came up with for all three catalogues demonstrates that in each case the choice of the actual stellar configuration used for the dating of the catalogue was conditioned by the Scaligerian dating of said catalogue's creation, whose veracity the authors of [273] and [274] were trying to prove. In other words, Y. N. Yefremov and Y. D. Pavlovskaya chose such stellar configurations for each catalogue in [274] that would concur best with the Scaligerian dating of the catalogue's compilation. A "method" such as this one is mere tailoring of research results in such a way that they would correspond to the desired values known a priori.

All of this makes the results claimed in [273] and [274] wholly insubstantial. These results are erroneous, and therefore cannot confirm the Scaligerian datings of the old star catalogues.

4.3. A vicious circle in the dating of the Almagest by the movement of the star o^2 Eri

Let us analyze the dating of the Almagest by the group of o^2 Eri as offered in the works of Y. N. Yefremov ([273] and [274]) in more detail, since it is this dating that Y. N. Yefremov bases his conclusions upon de facto.

We have already referred to the star o^2 Eri above, in section 1. Bear in mind that its identification as one of the Almagest stars is largely dependent on the alleged dating of the catalogue. In other words, the answer to the question of "who is who in the Almagest", or, in other words, whether the star o^2 from the constellation of Eridanus is represented in the Almagest at all, and if so, under which name, varies to a great extent as the a priori known dating of the catalogue changes.

Let us remind the reader that the star o^2 Eri moves fast enough, which changes its celestial position. In the course of its movement it becomes consecutively identified as different stars of the Almagest – namely, the three of them that one finds on the historical interval of the last 2,500 years. Bailey's numeration of these Almagest stars is as follows: 778, 779 and 780. The star #779 is traditionally identified as o^2 Eri (qv in [1339]) due to the mere fact that in the beginning of the new era the star o^2 Eri had occupied a position close to that of the star 779 on the Almagest star atlas.

However, what we face here is clearly an implication of the Almagest's being roughly dated to the beginning of the new era. If we are to make no presumptions in re the dating of the Almagest, we instantly find other candidates which we could identify as the moving star o^2 Eri. For instance, on the interval of 900-1900 A.D., the star which corresponds to the real position of o^2 Eri is #780. On the other hand, the star #779 from the Almagest does not remain unidentified in this case either, since it can be successfully identified as the star 98 Heis (see [1339], page 117). Furthermore, this is the exact identification of this star which was made by the astronomer Pierce, qv in [1339].

We must emphasize that the star o^2 from the constellation of Eridanus is rather dim, likewise the ones that surround it. Their magnitudes range from 4.2 to 6.3. Therefore, the only way of identifying them as Almagest stars is coordinate comparison. The brightness of these stars is roughly the same, and Ptolemy's verbal descriptions of the stars in this part of Eridanus are laconic and extremely vague. Therefore, a reliable identification of these stars by any other properties but their coordinates is impossible. The "proof" of o^2 Eri being veraciously identified as a star from the Almagest catalogue as cited in [274] is based on late identifications of the Almagest stars, or, alternatively, upon dating the catalogue to II century A.D. in actuality. The use of such "proof" for independent dating obviously leads us to a vicious circle.

Therefore, what we see in the works of Y. N. Yefremov and his co-authors ([273] and [274]) is in fact the assumption that the Almagest was compiled in the early days of the new era used as the basis for the corollary that the Almagest dates to 13 A.D. ±100 years. This is the very vicious circle that we're talking about.

4.4. Y. N. Yefremov's errors in the precision estimation of dating the Almagest by Arcturus

Let us now turn to Arcturus – the second and last star discussed in the work of Y. N. Yefremov and Y. D. Pavlovskaya ([273]). The Almagest identification of the Arcturus is unambiguous. The first proper motion dating of the Almagest that we encounter in [273] is 250 A.D. Then the authors "adjust" this dating and end up with the dating of 310 A.D. ±360 years calculated by one of the configurations. We shall deal with this "adjustment" below.

The dubiety of the results published in [273] and [274] was also commented upon by other authors. M. Y. Shevchenko, for one, makes the justified remark in re [273] that "the catalogue dates to the I century B.C.; however, the precision and hence the veracity of this result leaves much to be desired so far" on page 184 of [968].

Simple considerations allow for an easy estimation of the real precision that the method's leading principle is based upon (as related in [273]). Indeed, the Almagest position of a given moving star is determined in relation to certain stars in its vicinity ([273]) – the "Arcturus group" in case of Arcturus. The Arcturus group contains 11 stars. The position of Arcturus in relation to this group is used for the estimation of its position on the star chart theoretically calculated backwards for the epoch *t*. These positions are then compared to each other.

All the stars of the Almagest are measured with errors of some sort. This definitely applies to the "group" stars – in particular, all the stars from the group of Arcturus. Let us however make the temporary presumption that the measurements of the stars in the vicinity of Arcturus were carried out with ideal precision. Even in this case the error rate in the Almagest location of Arcturus cannot be less than 10' by any coordinate, since this is the grade value of the Almagest star catalogue's coordinate scale. In reality, this rate has to be raised due to the imprecise coordinates of the stars in a given group.

This leads to the arc distance error of circa 14' for [273]. If the possible error rate for each of the coordinates equals 10', it shall equal 14' for the hypotenuse according to the Pythagorean theorem. Proper movement speed for Arcturus is roughly 2" per year. Therefore, Arcturus covers the distance of 14' in about 420 years. This is but a rough estimation of the "method's" precision.

In reality, the actual precision of the position of Arcturus in the Almagest may be given with an error rate that substantially exceeds 14', and the dim stars in its vicinity could be measured with even less precision. What we are referring to here is naturally the arc distance error. As we shall see below, the latitude of Arcturus was measured with sufficient precision in the Almagest – however, this does not apply to its longitude (see Robert Newton's research in [614], for instance). Moreover, one has no reasons to assume that Ptolemy measured any of the dim group stars precisely. Therefore, the real precision of the "method" related in [273] is a lot worse than 420 years. Therefore the interval of possible datings of the Almagest obtained with this method is a priori known to be greater than 200 B.C. – 700 A.D.

Let us now comment upon the random error modelling method as offered in [273] and [274] for the precision estimation of the resultant dating. For instance, this "method" brought Y. N. Yefremov to the conclusion that his dating of the Almagest to roughly 300 A.D. had the precision of ±300-400 years (see [273], page 311, and [274], page 181).

The method of minimal squares is used for the purposes of dating in [273] and [274]. The elementary calculations cited above demonstrate the precision of this method to be estimated in accordance to the individual error rate pertinent to the Almagest position of the star under study divided by the speed of its proper movement.

Y. N. Yefremov uses the method of random modulation of the Almagest errors in order to raise the precision of his method. The precision of the modelling method that he suggests (multiple perturbations of the Almagest star coordinates resulting from the application of some random value "comparable" to the catalogue precision) isn't estimated anywhere in his works. Nevertheless, this method will only work if the results of these random perturbations shall make the Almagest stellar coordinates approximate the real ones with "distinctive" probability. However, due to the effect of the individual error mentioned above, the probability of such coincidence with the area of real coordinates shall most probably be very low. At any rate, this probability has to be estimated; there isn't so much as a hint of such estimation anywhere in [274]. In general, the methods offered by the authors of [273] and [274] don't hold water from the point of view of mathematical statistics.

The "dating modelling method" as offered by Y. N. Yefremov can be formulated in the following manner. One is to consider a certain vicinity of a fast star – Arcturus, for instance. Then one is to use the method of minimal squares in order to determine the date which gives us a minimal square average discrepancy of the mutual distance set of the Almagest stars from the set of the same values in the real stellar configuration that alters over the course of time. This dating is used for the estimation of the real date when the catalogue was compiled, which is unknown. Y. N. Yefremov marks said dating as T_0.

Furthermore, the resultant minimum of square average discrepancy is for some reason declared to be the dispersion estimation of the local error in the Almagest catalogue. Y. N. Yefremov tells us rather plainly that "grouping the same *n* quantity of stars in different ways, we shall obtain a number of estimations $\varepsilon_{\lambda,\beta}$. They aren't independent; therefore, instead of averaging them we shall choose the maximal value which shall be considered the estimation of the local coordinate determination error in the Almagest catalogue" ([273], page 311). One wonders just why. Firstly, the local error of the Almagest has to be estimated separately, which is necessary for the understanding of just what minimal level variation we must allow for in order to reliably cover the real dating of the catalogue's compilation. When Y. N. Yefremov takes the actual minimal value for dispersion estimation, he basically fails to allow for the variation of this minimum altogether.

Secondly, the sample volume used for the averaging of the value in question is too small (circa 5-6 independent observations) and doesn't permit to consider Y. N. Yefremov's estimation precise enough. Local error needs to be estimated from a much greater quantity of stars.

Furthermore, Y. N. Yefremov models random perturbations of Ptolemy's coordinates using his "estimated" local error rate as basis. He writes that "the knowledge of the error rate $\varepsilon_{\lambda,\beta}$ for each group makes it feasible to conduct a numerical experiment in order to study how the estimation of T_0 is affected by random coordinate errors. Let us model the corrections of stellar coordinates from the Almagest catalogue, considering these corrections to be distributed normally with the average of zero and the square average error $\varepsilon_{\lambda,\beta}$ for each group and calculate the respective value of T_0. Having repeated the procedure 100 times, we can build a distribution graph for the resultant estimations of T_0" ([273], page 312). Y. N. Yefremov proceeds to tell us that "the common interval for all the groups with the square average errors for the epochs of $\overline{T_0}$ taken into account is the I century B.C." ([273], page 313). Y. N. Yefremov also makes the following flabbergasting statement: "the probability rate of T_0's random value exceeds 900 equals 0.2, and that for a group with maximal dispersion. Therefore, the Almagest catalogue is most unlikely to be a mediaeval forgery" ([274], pages 188-189). Thus, Y. N. Yefremov apparently assumes that the average date must be close enough to his "randomly modelled date" $\overline{T_0}$, estimating this proximity whilst "taking the square average errors as calculated above into account" ([273], page 313).

This approach is utterly delusional. It is obvious that what Y. N. Yefremov determines to be the average modelled date $\overline{T_0}$ is merely his initial estimation of T_0 with some random perturbation added thereto by the author himself. As for the distribution of modelled dates, what he comes up with is a random dispersion with the centre equalling T_0 for a given group. Y. N. Yefremov is of the opinion that the real date must be close to the centre of this dispersion, or, in other words, that the random perturbations that he introduced have a certain real probability of covering the real positions of Ptolemy's stars. In other words, he hopes that his modelling will randomly cancel out Ptolemy's errors, estimating their probability to be real. This is the exact meaning of the passage quoted above where Y. N. Yefremov tells us that a post-900 A.D. dating can only be achieved in the course of this modelling with "the minute probability rate of 0.2". He is of the opinion that this makes a mediaeval dating of the Almagest highly improbable.

However, one has to bear in mind that his initial dating T_0, which the modelled datings are grouped around differs from the real date by a certain value. The value of this shift, as we have demonstrated above by simple calculations, can be great enough. In case of Arcturus its lowest possible value is 420 years, qv above. Said shift is defined by Ptolemy's individual error in the estimation of a given star's coordinates, as well as individual errors for the stars of the chosen group. Also, our calculations demonstrate that the value in question is largely dependent on the group choice. Therefore, some individual error is already inherent in the value T_0, possibly a serious one. When Y. N. Yefremov "models" his additional errors for group stars, he already distributes them around a certain dating which might be shifted sideways to a substantial degree. However, in his reference to the graphs of modelled distributions, Y. N. Yefremov appears to assume that the real dating must be located near the centre of these distributions in every case – at least, within a certain confidence interval with the probability ratio of 0.8, since he considers the probability of 0.2 to be too low.

This is untrue. The abovementioned simple estimation demonstrates the real date to be far enough from the centre of such modelled distribution (for instance, this range exceeds 420 years for Arcturus, qv above). At the same time, the scatter range of modelled dates around a shifted date might not be all that great. The matter is that Y. N. Yefremov takes an unreasonably low value of the square average error obtained from parabolic minimum for this modelling, making no specific estimations of this error for some reason.

Apart from that, it is easy enough to estimate that even if one is to model the correction for the coordinates of a single star, the probability of returning to its true position is very small in general. This is confirmed by the following simple calculation. Let us assume that Ptolemy's individual error for a given star equals 45 arc minutes. Such errors are typical for the Almagest – a great number of stars it contains were measured a lot worse ([1339]). Let us re-emphasize that we are referring to the arc error. Latitudinal errors are a lot smaller, as we shall demonstrate below.

If we apply the above calculations to Arcturus, for instance, the implication is that in order to model an actual dating that would differ from the original by 400 years maximum, one has to "hit" the 14-minute range around the star's real location (provided that the group stars have already fallen into necessary positions and do not affect the dating too greatly). The maximum probability of the value falling into this 14-minute range from a position shifted by 45' can be estimated as the probability of its falling into the shaded sector on fig. 3.13a.

If we are to consider the probability of a perturbed point being located in the 60' radius of point A to equal 1, we end up with the probability of 0.1 for its location in the shaded sector. Thus, even in this ideal case the probability rate of obtaining the necessary dating randomly – not even the correct dating, but rather one that won't differ from it by more than 400 years, equals 0.1. Still, Y. N. Yefremov is of the opinion that the probability threshold of 0.2 already suffices for rejecting the post-900 A.D. datings as improbable.

The authors of [274] claim that the results of calculations performed by other fast stars (which aren't cited in their work for some reason) confirm the conclusions made in the research of Arcturus and o² Eridani. However, this statement does not correspond to reality.

Let us provide a single vivid example. Among the fast stars which were processed by the authors of [273] and [274] we find Procyon, a star which was famous in mediaeval astronomy. Our research (qv in section 1, for instance) demonstrates that Y. N. Yefremov's method must have led to the dating of roughly the X century A.D. by Procyon, which would blatantly contradict his conclusions. For a mysterious reason, [273] tells us absolutely nothing about the results for Procyon.

Finally, the "method" related in [273] and [274] is largely dependent on the group contingent choice for the fast star under study. We have checked how the result of the dating by the Arcturus group changes depending on the choice of various stars for this group. It turns out that when we change the contingent of the group, the Arcturus dating may vary from 0 A.D. to 1000 A.D. – that is, the results can fluctuate with the amplitude of up to a thousand years. This very circumstance completely invalidates the method offered by Y. N. Yefremov.

Corrolaries:

1. The result of dating the Almagest by proper star motions as claimed by Y. N. Yefremov and Y. D. Pavlovskaya in [273] and [274] is based on thin air. Furthermore, some of the considerations one encounters in said works contain a "vicious circle".

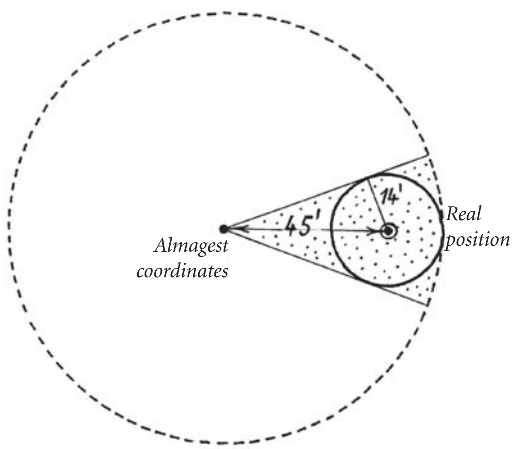


Fig. 3.13a.

2. If we are to strip the works in question ([273] and [274]) from all such "circular" considerations, the "discrepancy" we end up with does not contradict our dating, qv below.

3. The positions of Y. N. Yefremov and Y. D. Pavlovskaya that concern the precision estimates of their method (and the correction modelling of the Almagest) as seen in [273] and [274] are mathematically illiterate and void of meaning in our opinion.

4. The authors of [273] and [274] failed to consider Procyon, which gives a blatantly non-Scaligerian dating, for some "unknown reason".

The work of Y. N. Yefremov and Y. D. Pavlovskaya ([273]) was published in the "Doklady Akademii Nauk SSSR" in 1987. We pointed out the errors contained in [273] and [274] in our articles [350] and [355], which were published in the "Doklady Akademii Nauk SSSR" in 1989 and 1990, respectively. Apart from that, we have personally addressed Y. N. Yefremov with a criticism of his errors at the seminar hosted by the Institute of Natural Scientific and Technical History in 1989. Y. N. Yefremov did nothing to rectify the errors in question – moreover, he evades all attempts of their discussion.

4.5. Erroneous precision estimation of astronomical calculations: another example

Let us consider another publication that deals with the issue of Almagest dating ([179]). Its authors, Y. S. Goloubtsova and Y. A. Zavenyagin, refer to Galley reporting that over the time that passed between Ptolemy and Galley (up to 1690, which is when Flamsteed's star catalogue was created), Arcturus shifted in the Virgo direction by 1.1 degrees. Having compared this to the annual shift value for Arcturus (2.285"), Goloubtsova and Zavenyagin perform the following simple calculation, writing that "if we are to divide 1.1 degrees by 2.285 angular seconds per year, we end up with 1733 years. Finally, once we subtract 1733 from 1690 (or the year when Flamsteed's catalogue was compiled), we shall come to the conclusion that the Almagest catalogue was compiled in 43 B.C. The discrepancy error rate for the coordinates of neighbouring stars is a lot smaller than the error of the actual coordinate, since the subtraction removes the systematic error. Therefore, the average error rate in the positions of bright stars in relation to their neighbours in the Almagest does not exceed 0.1 degrees [? – Auth.]. The implication is that the possible dating error rate does not exceed 150 years" ([179], page 75).

Thus, if the authors of [273] date the catalogue to 250 A.D. by Arcturus (and even to 310 A.D. after making their "adjustment", estimate precision equalling ±360 years in this case), the authors of [179] perform a single solitary arithmetical calculation and date the Almagest to 43 A.D., also by Arcturus, with the much greater precision rate of ±150 years.

However, the text from [179] as quoted above is oriented at the reader who will not bother checking the real stellar configuration on the celestial sphere. The calculations of the authors of [179] are based on the taciturn implication that the own movement vector of the modern Arcturus is directed exactly at its Almagest location. Had this indeed been the case, their calculations would have some sort of reasoning to back them up. However, this doesn't appear to be the case. In fig. 3.1 one sees the real movement direction of Arcturus in relation to its position as specified in the Almagest. One can plainly see that Arcturus moves visibly "sideways" from its Almagest position. Therefore, it isn't the value of 1.1 degrees that has to be divided by 2", the way it is done by the authors of [179] for some reason, but one that is a great deal smaller, and shall yield the dating of approximately 900 A.D., albeit with a significant possible error rate due to the rough nature of the method itself. See our considerations in re the precision of this method above.

Thus, dating the Almagest to 43 A.D. with the possible discrepancy rate of ±150 years, as Y. S. Goloubtsova and Y. A. Zavenyagin claim to have done, is completely out of the question.

Let us also point out that the very "concept" behind [179], which implies the random errors in the Almagest to be a result of proper star movement, is perfectly erroneous. Its absurdity is all the more obvious if we are to consider the examples of slowly moving stars which are almost immobile. The division of a non-zero error of the Almagest in the position of a star might yield any "infinitely ancient" observation dating.

The claim made by the authors of [179] in re the error in the bright stars' positions in the Almagest not exceeding 0.1 degrees, or 6', isn't based on anything whatsoever. Why 6' and not 2' or 15'? Having said everything about the precision estimation problem of the Almagest stellar coordinates, we deem a deeper study of this issue superfluous.

The authors of [179] did not limit their research to the study of Arcturus and its behaviour. They also attempted to date the catalogue by another "fast" and well-known star – Procyon. Let us quote: "We get a similar result once we date the Almagest by the proper movement of Procyon, namely, that the Almagest catalogue was compiled in 330 B.C., with the possible error rate of ±300 years… The Procyon dating serves as a perfectly independent corroboration of the Arcturus dating, both of which take us to the last centuries before the new era" ([179], pages 75-76).

However, just as they had done in case of Arcturus, the authors did not take the direction of Procyon's movement into account for some reason. Let us see what "dating" we shall get if we are to use their "method" for our own accurate calculations which take real stellar positions into account. It turns out that the real trajectory of Procyon's movement is such that a rough Procyon dating is the X century A.D., no less (see section 1). It goes without saying that the issue of this dating's precision remains standing.

4.6. The "secondary analysis" of the Almagest dating in the "Samoobrazovaniye" ("Autodidactics") magazine

In the first 1999 issue of the Muscovite magazine "Samoobrazovaniye" ([263]) we find a publication by A. S. Doubrovskiy, N. N. Nepeyvoda and Y. A. Chikanov entitled "On the Chronology of Ptolemy's 'Almagest'. A secondary mathematical and methodological analysis" which deals with our dating of the Almagest by proper star movements in particular.

Unfortunately, the authors of [263] failed to familiarize themselves with the necessary astronomical issues and thus made the false conclusion that the dating of the Almagest by proper star movements is unreliable in general, as the speeds of proper star movements are known rather badly, which is presumably reflected in great controversy one finds in astronomical literature.

Further in [263] we encounter a comparative table of proper movements as taken from the "Astronomicheskiy Yezhegodnik" ("The Astronomical Yearly") and the catalogue [1197]. For instance, the reader is invited to compare the values contained in both catalogues (-0.1098; -0.2001) and (-1.155; -1.998) respectively. These are the proper movement speeds of Arcturus.

The authors of [263] tell us exactly the following in this respect: "As for the analysis of the "fast" star motion, we must point out that the data concerning the stellar speed taken by Fomenko's group from the catalogue… [followed by a reference to the bright star catalogue ([1197]) – Auth.] differ considerably from those contained in the "Astronomicheskiy Yezhegodnik" ([263], page 23).

Having cited this remarkable table on page 24 of [263], its authors come to the following conclusion: "As one sees from the table, estimating the age of the catalogue by proper star movements is a more than dubious activity which doesn't stand up to criticism". However, the speed vector compounds which are compared in this table weren't just given in different coordinate systems, but also in different measurement units! This is easy to observe from the above example – we're dealing with the equatorial coordinate system for the epoch of 2000 A.D. in one case and the equatorial coordinate system for the epoch of 1900 A.D. in the other. These coordinate systems differ from each other. The above example demonstrates the scale discrepancy. According to the Pythagorean theorem, the given vector speed components of Arcturus suffice for the calculation of said vector's length which shall already be independent from the coordinate system. However, in the first case it is ten times smaller than in the second, which stems from the fact that different catalogues use different proper movement scales. In one case the measurement unit used equals 1/1000th of a second per year, and in the other it is 1 second per century. The units differ by a factor of ten.

One needs no commentary here. It is obvious that before suggesting that the reader should compare any values of any kind, said values need to be given in the same scale.

We shall refrain from discussing the authors' own attempts of dating the Almagest ([263]), merely stating that we are of the opinion that the dating of the Almagest has to be preceded by an in-depth study of certain rather complex issues from the part of the researcher. It actually requires a great deal of time and effort, even from a specialist.

5. CONCLUSIONS AND DIRECTIONS FOR FURTHER RESEARCH. OUR APPROACH AND A BRIEF SYNOPSIS OF OUR MAIN RESULTS

5.1. The three problems one is confronted with: identifying the Almagest stars, defining the nature of possible errors, and analysing the precision of the catalogue

Sections 1-3 contain accounts of several attempts to date the Almagest on the basis of the numerical material contained in Ptolemy's star catalogue. All of these attempts has proven futile. We have discussed them in such great detail for two reasons – firstly, the reader can get a better idea of what the complexities of the "self-sufficient" dating of the star catalogue really are – the dating that would be based on nothing but the catalogue's numerical material, that is. Secondly, we wanted to provide some basis for raising the issues that we shall relate in more detail further on.

The main corollary that we come to at the present stage is as follows. The dating of the Almagest requires a meticulous preliminary analysis of the catalogue. This analysis must relate to the following issues.

1. Identifying the Almagest stars as the ones observed on the contemporary celestial sphere. In section 1 we demonstrate that this problem doesn't always have an unambiguous solution; furthermore, the solution in question might depend on the alleged dating of the catalogue. Therefore, before we can proceed with dating, we have to find and reject all cases of dubious identification of the Almagest stars as their modern counterparts.

2. The nature of possible errors contained in the Almagest catalogue. The error rates in stellar coordinates characteristic for the Almagest lead one to the conclusion that the dating of the catalogue cannot be estimated with more precision on the historical interval as based on proper star movements. However, this statement becomes generally false if we manage to discover the systematic compound in the errors of

the Almagest star positions. In this case we may get an opportunity to compensate it, thus raising the precision of the catalogue, which, in turn, may allow us to date the latter regardless of the error in question.

3. The precision of the Almagest catalogue attained with different stellar subsets. The goal of this analysis is the choice of the star group from the Almagest whose coordinates must have been measured by Ptolemy with some guaranteed precision level δ. Once we manage to locate such a group, it shall define the set of possible Almagest datings, namely, making feasible the datings that will allow the guaranteed precision level δ to be attained for the stars of this group. If the resultant dating interval proves to be a great deal shorter than the a priori known historical interval, we shall obtain purposeful information about the date when the Almagest star catalogue was compiled. This concept shall be used below (see Chapters 5-7).

Let us briefly discuss each of the three issues as listed above. Their more detailed rendition can be found in the chapters to follow.

5.2. The identification of the Almagest stars

There is a rather large amount of handwritten copies as well as several mediaeval printed versions of the Almagest where the ecliptic coordinates of individual stars differ from one another. Most of these copies and editions (although not all) were brought to roughly 60 A.D. by precession. The implication is that if one were to compare the stellar longitudes from a given copy of the Almagest with the precisely calculated stellar longitudes for 60 A.D., the average discrepancy rate shall equal zero. Such a comparison is only possible due to the fact that identifying most of the Almagest stars with those on the modern celestial sphere leaves no room for doubt.

The source text that we used was the Almagest catalogue containing over a thousand stars in the exact same form as it is given in the fundamental work of K. Peters and E. Knobel ([1339]). Several coordinate variants from [1339] were also included in the list of stars under analysis. In the preliminary stage we neither doubted the veracity of stellar coordinates from the Almagest, nor the fact that they were given in ecliptic coordinates rendered to 60 A.D. due to precession.

As it has already been mentioned, [1339] contains the identifications of the Almagest stars as their modern counterparts. Nevertheless, we have conducted the identification process from scratch in order to select the stars to be analyzed, see Chapter 4. The identifications contained in [1339] were thus confirmed for the most part.

However, we have discovered several modern stars that can be identified as different Almagest stars for different epochs *t*. Such are o^2 Eri and μ Cas, for instance. These stars were identified in [1339] under the assumption that Ptolemy's observations were conducted around the beginning of the new era. Basing the dating of the Almagest catalogue on the analysis of such stars makes no sense, for we shall simply end up with a vicious circle. All such stars were excluded from further consideration.

Let us also point out that the identifications and coordinates of the stars o^2 Eri and μ Cas are considered doubtful.

5.3. Various types of errors in the catalogue

We have demonstrated above that a simple comparison of the calculated stellar coordinates to those contained in the Almagest catalogue doesn't permit to estimate the dating of the latter. This is explained by the huge discrepancy rates inherent in the Almagest catalogue for the most part. Therefore, we can only succeed if we analyze the Almagest errors of different nature meticulously.

We shall divide the errors into three types: group errors, random errors and "rejects".

Under group errors we shall understand various data distortions resulting from observations or recalculations and leading to the shift of a star group on the celestial group as a whole.

Random errors are of an individual character and owe their existence to imprecise observations ranging within the grade value of the measurement instrument for the most part. A distinctive trait of such errors is that they shift each star on the celestial sphere by a random value which has a zero average.

Rejects are a product of circumstances which were either unforeseen by the compiler or unknown to him: copy errors, refraction etc. They also affect the coordinates of individual stars, and their values are usually much greater than the measurement instrument scale precision. Rejects are a rather scarce type of error.

The most important task is to define and compensate the group errors. Suitable methods are discussed in Chapter 5 where, apart from providing the formulae necessary for their calculation, we also demonstrate how to determine the precision of the resulting values.

The estimation of different types of errors in the Almagest stellar coordinates is dealt with in Chapter 6. We find out that the coordinates of stars as given in the Almagest do indeed contain significant group errors manifest as the shifts of the respective stellar configurations on the celestial sphere as a whole.

The values of group errors may in fact differ for various stellar groups – constellations, for instance, hence their name. However, we shall witness that insofar as large enough celestial areas are concerned, group errors of the Almagest and other old star catalogues coincide for various constellations and equal the single error for the entire area. We shall refer to such an error as the systematic error of a given catalogue for a given celestial area.

Each of the shifts defining a group error can be described by three parameters. We shall choose the following base errors as such, qv in fig. 1.1, Chapter 1.

Error τ in the location of the vernal equinox point $Q(t_A)$ made by the observer in the observation year t_A in the ecliptic direction. In other words, τ is the projection of the Almagest catalogue vernal equinox point shift sideways from its real position over the ecliptic.

Error β in the location of point $Q(t_A)$ in the direction of the meridian, or the projection of the error vector over the ecliptic meridian.

Error γ in the angle ε between the ecliptic and the equator. The change of a star's ecliptic coordinates by the ground observer needs to be preceded by the estimation of the angle ε between the ecliptic and the equator, regardless of the measurement method. If the observer made the error γ in the estimation of said angle, the ecliptic of the catalogue shall be shifted in relation to the position of the real ecliptic in the observation year by the value of γ.

The possibility that group errors may be inherent in the Almagest has been discussed by many researchers – see [1339], [614] and [544], for instance. We shall merely mention possible reasons for the existence of such errors here.

Error τ might result from the fact that the observer or a later compiler of the catalogue had for some reason "adjusted" the catalogue to make it fit a dating that would differ from that of the real observation. It is possible that this operation used to serve some methodological end – for instance, making the catalogue conform to some round or important date. It could also have been used for a deliberate distortion of the real observation date ([614]), or, alternatively, it may result from changes in the initial longitudinal reference point. We have already demonstrated that ancient astronomers could count longitude from various points on the ecliptic. A change of the initial reference point would naturally lead to some constant being added to all ecliptic longitudes and hence the alteration of the catalogue's "dating", if it were to be dated by longitudinal precession.

It is understandable that the latitude of a star is independent from error τ. This makes latitudinal coordinates more reliable, which is the very reason why we shall be considering longitudes and latitudes separately. The consideration of latitudinal discrepancies requires just two parameters to define a group error – β and γ, for instance.

What is there to say about the values of β and γ? Equatorial latitudes of stars are easy enough to determine from actual observation with enough simplicity and precision ([75]). Therefore, one should expect error β to be small enough for the moment of observation, provided the observer was accurate enough. Error γ is of a principally different character. The determination of the ecliptic position is achieved as a result of rather complex observations and calculations, qv in Chapter 1. Therefore, the value of error γ might be significantly greater than that of error β.

The works [544] and [1339] contain indications at the fact that the systematic error γ is indeed inherent in the Almagest. Moreover, some of the Almagest's researchers estimated the value of this error as roughly 20'. Our calculations confirm this, qv in Chapter 6.

We shall occasionally use parameters φ and γ instead of β and γ since they are more convenient from

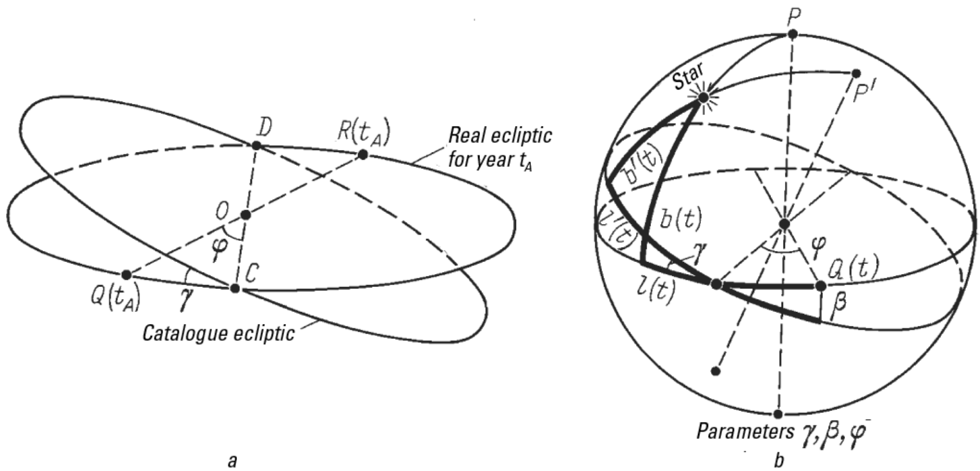


Fig. 3.14. Specifying the parameters of the systematic error in the ecliptic coordinates of the stars with the aid of the parameters γ and φ or γ and β. In the present example τ = 0.

the point of view of calculation. Their meaning is clarified in fig. 3.14. Inasmuch as the latitudinal discrepancies are concerned, the group error is rendered to a mere misplacement of the ecliptic plane, which we shall be referring to as the "catalogue ecliptic". One can define the mutual disposition of the catalogue ecliptic and real ecliptic plane for catalogue compilation epoch t_A if one is to fix angle φ between the equinox axis *QR* for epoch t_A and the plane rotation axis *CD*, as well as fixing the plane angle γ between the two ecliptic planes – the true and the false. We shall hereinafter define the parameters of group errors with the values of φ and γ for the most part.

Generally speaking, the compiler of the catalogue may have made different group errors in his study of different celestial areas. Possible reasons include instrument readjustment, the choice of a different observation point etc.

In Chapter 2 we discover seven parts of the Almagest star catalogues which are naturally distinctive as seen on the celestial sphere, and differ by their reliability characteristics in the Almagest, see fig. 2.14. In Chapter 6 we shall see that the same celestial areas in the Almagest also differ in group error values and precision characteristics.

To sum up, one can say that the reasons for the existence of group errors and other discrepancies as listed above only serve to explain the possible mechanisms of error genesis. Calculations allow the discovery of errors themselves but tell us nothing of how and why they were made – possible reasons may differ from the abovementioned.

5.4. The discovery of the systematic error in the Almagest catalogue. Its compensation confirms the correctness of the declared catalogue precision

The real moment t_A of the catalogue's compilation remains unknown to us. Therefore we should calculate the values of parameters $\gamma(t)$ and $\varphi(t)$. The calculation method is a combination of the minimal square method and the spherical regression problem. Its precision properties are discussed in Chapter 5.

The results of our calculations can be represented as graphs $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$, qv in fig. 3.15. These graphs were built after the processing of the Almagest stellar coordinates for large celestial areas. The "stat" index indicates that the corresponding values were educed by methods of statistics. They are actually estimates of discrepancy parameters inherent in the positions of the Almagest stars, and demonstrate said discrepancy to be uniform for several large areas of the celestial sphere. The estimations were made under the assumption that the catalogue was compiled in epoch *t*, and are thus *t* functions. We shall be using the term "systematic errors" for the error in question as well as its compounds, parameters $\gamma(t)$ and $\varphi(t)$.

What is the relation between these errors and group errors? If the large celestial area under study consists of several constellations, systematic errors discovered with the aid of statistical methods shall represent averaged group error values for different constellations. It is only in case when all group errors equal each other that they coincide with the respective systematic error.

This is the only case where we shall not differentiate between the definitions of "group error" and "systematic error".

We have built confidence intervals I_γ and I_φ of acceptable γ and φ values around each value of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$. Let us clarify that γ_{stat} and φ_{stat} are but punctual statistical estimations of unknown parameters; the latter define the systematic error made by the compiler of the catalogue, and the values of such estimations are by no means equal to the values of actual unknown parameters. Once we build the confidence intervals around the calculated punctual estimates γ_{stat} and φ_{stat}, we can claim the true parameter values to fall into these intervals with a given degree of certainty.

The method of building confidence intervals, which is widely used in statistical problems, is related in Chapter 5. Actual results pertaining to the Almagest are cited in Chapter 6.

We have conducted an analysis of errors for all seven celestial areas of the Almagest as discovered above, having determined their respective systematic error values as well as the values of the "remaining" square average latitudinal discrepancies resulting from the compensation of the discovered conditional systematic errors. What we discovered as a result was that areas *A* and *Zod A* are the most precisely-measured of all, qv in Chapter 6 and table 2.3. A propos, these are the areas where most of the named Almagest stars are located. Another discovery was that after the compensation of systematic error, more than half the stars from area *A* ended up with the latitudinal discrepancy of 10' maximum (see Chapter 6). The percentage of such "well-measured" stars is even greater for area *Zod A* – 63.7%. Thus, the declared 10' precision rate of the catalogue was confirmed for the latitudes of the majority of stars from a rather large celestial area.

The next issue that we are confronted with is the nature of the discovered parameters γ_{stat} and φ_{stat}. Is it true that the calculated values of γ_{stat} and φ_{stat} are close enough to real group errors for the entire catalogue, or at least the stars from area *A*?

It is quite possible that the compiler of the catalogue made individual group errors for each constellation; in this case, the values that we have calculated shall de facto represent a sum of various averaged group errors, the result of such averaging being non-zero due to the relatively small number of constellations in general.

In order to answer this question, we have considered all the Zodiacal constellations and the "neighbourhoods" of most named stars. Calculations have shown that the value of $\gamma_{stat}^{Zod\,A}$ as calculated for area

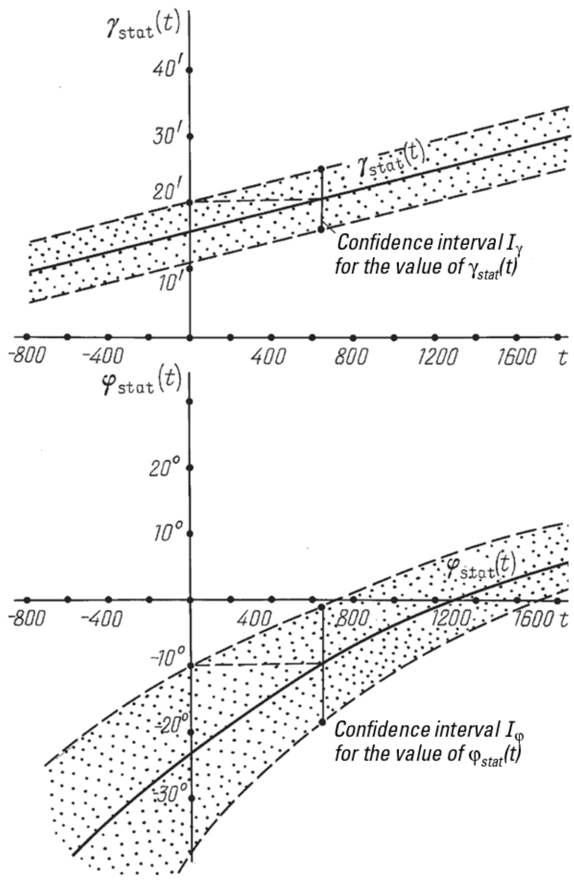


Fig. 3.15. The behaviour of parameters $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ in time.

Zod A applies to all the constellations from area *A* at least. In other terms, $\gamma_{stat}^{Zod\,A}$ should be regarded as the systematic compound that affects all the stars from the well-measured celestial area *A* which also contains most of the named stars. However, we can make no such claim for the value of $\varphi_{stat}^{Zod\,A}$. It is curious that this conclusion about the nature of compounds γ_{stat} and φ_{stat} can serve as argumentation in favour of the theory that the coordinate measurements for the Almagest catalogue were conducted with the use of the armillary sphere. See Chapter 6 for more details.

5.5. The compensation of the systematic error discovered in the catalogue gives us an opportunity of dating the latter

The compensation of the discovered systematic error allowed us to reduce the latitudinal discrepancy for area *Zod A* of the Almagest from 17.7' to 12.8'. This resulted in the possibility of dating the catalogue.

We have already pointed out that the declared 10-minute precision rate of the Almagest is indeed attained for most of the stars in the catalogue. The question that one comes up with here is whether there are any stars at all for which the declared Almagest precision rate will be guaranteed?

It is known that the observer always uses the system of referential points, or stars, on the celestial sphere in stellar coordinate measurements, qv in [968], for instance. This measurement method is natural and has been used by all mediaeval astronomers. Tycho Brahe, for one, used 21 referential stars for his measurements ([1049]). The modern system of referential points consists of several thousand stars which are collected in the so-called fundamental catalogues (see catalogue FK4, for instance – [1144]). The Almagest contains indications that Regulus and Spica must be among these referential stars. Special sections of the Almagest are dedicated to the measurement of their coordinates.

Let us formulate the following axiom. If the declared precision of the catalogue is confirmed, it should be guaranteed for the majority of the referential stars from the catalogue in question.

What are the stars that should have necessarily been included in the number of the Almagest's referential stars? First and foremost, Ptolemy must have used those of the stars which have names of their own in his catalogue. There aren't too many such stars – only twelve. They really comprise a very convenient basis in the visible part of the sky. Their complete list is as follows: Arcturus, Regulus, Spica, Previndemiatrix, Capella, Lyra = Vega, Procyon, Sirius, Antares, Aquila = Altair, Aselli and Canopus; twelve stars altogether.

All of these stars are bright and clearly visible against their background. What is especially important for the purposes of dating, some of them have a rather high proper movement speed – for instance, Arcturus, Procyon and Sirius. Some of the others also shift across the celestial sphere rather visibly, namely, Regulus, Capella, Antares and Aquila = Altair.

However, we had to exclude two of the twelve stars from consideration instantly – namely, Canopus and Previndemiatrix, the reason being that Ptolemy's coordinates of Canopus were greatly affected by refraction, and they can be regarded as a "reject" from the statistical point of view; as for Previndemiatrix, Ptolemy's initial coordinates of this star were lost, and simply remain unknown to us today, qv in Chapter 2.

Two more stars (Sirius and Aquila, or Altair) were rejected due to the fact that the systematic error is different in their case, as our analysis shows, and the value of said error cannot be determined for these two stars. Therefore, the dating of the Almagest catalogue was made on the basis of the remaining 8 named stars. Their list is as follows:

Arcturus, 16, α Boo, Bailey's Almagest number 110;
Regulus, 32, α Leo, number 469;
Spica, 67, α Vir, number 510;
Capella, 13, α Aur, number 222;
Lyra = Vega, 3, α Lyr, number 149;
Procyon, 10, α CMi, number 848;
Antares, 21, α Sco, number 553;
Aselli, 43, γ Cnc, number 452.

5.6. The dating of the Almagest catalogue by the motion of its eight primary basis stars after the rectification of the statistically discovered catalogue error

The proposed hypothesis leads us to the implication that for the desired catalogue compilation epoch t_A, all of the eight named basis stars of the Almagest must have a maximal latitudinal discrepancy of 10'.

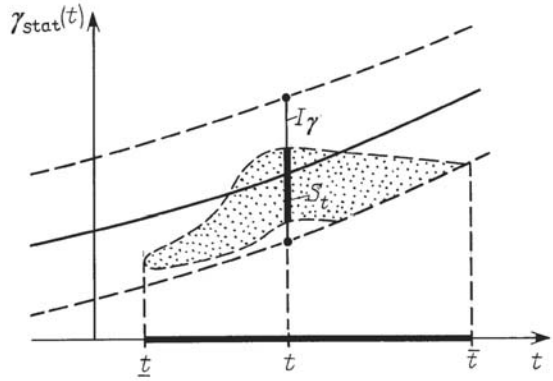
On the other hand, we know that the catalogue's systematic discrepancy compound γ must fall into the confidence interval I_γ built around the statistical estimation $\gamma_{stat}(t_A)$ for epoch t_A. We thus come to a natural dating method.

Let us consider the confidence interval I_γ around $\gamma_{stat}(t)$ with the value of t and the level of confidence being fixed and select a certain subset S_t from values that fall into it, which will compensate the given systematic error compound γ and make the latitudinal discrepancies for all of the eight named basis stars less than 10', or the grade value of the Almagest catalogue coordinate scale, with γ in S_t, qv in fig. 3.16.

In general, set S_t can be empty. Let us find all the values of the presumed datings t for which the sets S_t are not empty. These very values shall comprise the possible dating interval, since for all of the presumed datings t from this interval the latitudes of all eight named stars are measured with the precision rate of 10'.

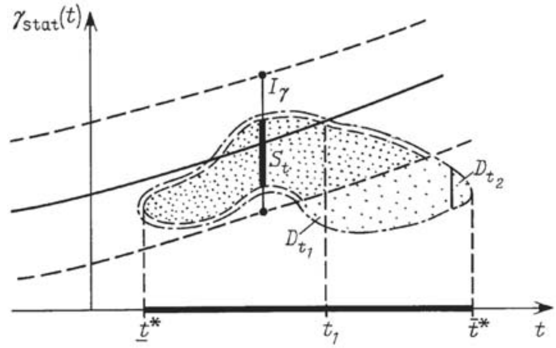
We shall refer to the described dating procedure as "statistical", since it is based on the values of $\gamma_{stat}(t)$ discovered with statistical methods. A more explicit description of this procedure can be found below, in Chapter 7, alongside a detailed discussion of the achieved dating results.

It turns out that the dating interval begins in 600 A.D. and ends in 1300 B.C. Although its length equals 700 years due to the low precision of the Almagest, this interval is located at a considerable distance from the Scaligerian dating of the Almagest's creation.



Possible dating interval

Fig. 3.16. Dating the Almagest catalogue with the statistical method.



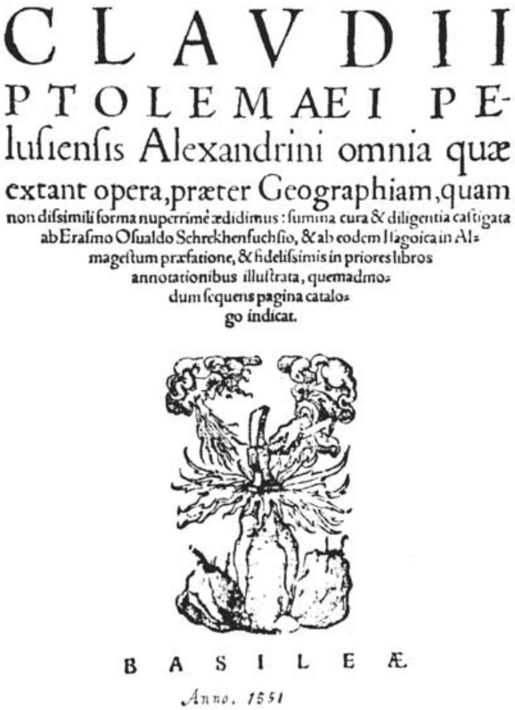
Possible dating interval

Fig. 3.17. Dating the Almagest catalogue with the geometrical method.

5.7. The dating of the Almagest catalogue by the motions of its eight named basis stars by an independent geometrical method

The confidence intervals used for the statistical procedure contained a certain subjectively chosen parameter, namely, the level of confidence, which represents the minute probability which we can disregard in statistical corollaries. Therefore one can actually discuss the issue of the dating interval being dependent on the chosen level of confidence. Our corollary that the group error for the 8 named stars equals the systematic error for area *Zod A* is also of a statistical nature and may therefore prove incorrect. Hence the question of just how much greater the discovered interval can become if the confidence areas expand indefinitely.

We shall give a "geometrical" answer to this question. Let us once again select a fixed time moment t as a candidate for the desired dating moment. After that we shall define the set D_t of such γ values that a turn of the real ecliptic by this angle for epoch t shall make the latitudinal discrepancy of all the 8 named stars conform to the 10-minute threshold with a certain value of parameter φ, qv in fig. 3.17. It is obvious that D_t contains subset S_t whatever the value of t might be. Therefore, we shall discover all the possible values of t for which the latitudes of all 8 named stars shall not differ from the respective stellar lati-


CLAVDII
PTOLEMAEI PE-
lusiensis Alexandrini omnia quæ
extant opera, præter Geographiam, quam
non dissimili forma nuperrimè ædidimus: summa cura & diligentia castigata
ab Erasmo Osualdo Schrekhenfuchsio, & ab eodem Isagoica in Al-
magestum præfatione, & fidelissimis in priores libros
annotationibus illustrata, quemadmo-
dum sequens pagina catalo-
go indicat.

BASILEÆ.
Anno. 1551

Fig. 3.18. The title page from a 1551 edition of the Almagest. The handwritten dating "Anno 1551" is most noteworthy indeed; the book is likely to have been dated retrospectively, in the XVII-XVIII century.

tudes as given in the Almagest by more than 10' after a certain rotation of the ecliptic.

A most important fact is that the resultant maximal possible geometrical dating interval coincides with the interval discovered by statistical methods. See Chapter 7 for more details.

Another fact that we shall demonstrate in Chapter 7 is that the proposed dating method possesses a certain stability unaffected by the variation of the initial hypotheses, the declared precision of the catalogue, the reduction or expansion of the dating contingent of the referential stars, and also the non-linear measurement instrument distortions.

The viability of our method has also been tested on the star catalogues compiled artificially as a result of modelling random errors in stellar coordinate observations. The "observation dates" defined in modelling concur with the results of dating by our method in every case.

Apart from that, the dating method that we offer was successfully tested on several well-known old catalogues. We have used it for dating the catalogues of Ulugbek, Al-Sufi, Tycho Brahe and Hevelius. In every case the traditionally known datings of the old star catalogues under study were confirmed with our methods, the Almagest catalogue being the sole exception. This is apparently an indication that the traditional dating of Ptolemy's lifetime contains a gigantic error of several centuries or even over a millennium. See Chapter 9 for more details.

Our main corollary is as follows. The star catalogue of the Almagest was created in the interval between 600 A.D. and 1300 A.D. The Scaligerian dating of the Almagest catalogue (II century A.D.) is ipso facto proven gravely erroneous.

We shall conclude this chapter with citing the front page of a 1551 edition of the Almagest (see fig. 3.18). It is most curious that the publication date is written by hand, in the exact same place of the book's front page where one expects to find a printed date. It is possible that this date was inscribed on the book as late as the XVII or even the XVIII century, possibly with the goal of making the book seem published in the XVI century, its real publication date being much more recent.