CHAPTER 6

Statistical and precision-related properties of the Almagest catalogue

1. INTRODUCTORY REMARKS

In the preceding chapters we have estimated that one of the primary problems with the dating of the Almagest by proper star motions is the problem of real precision of the Almagest catalogue star latitudes for different celestial regions. Therefore, one needs to conduct a meticulous analysis of star coordinate errata in the catalogue in general and different parts of the latter. A preliminary and rather rough analysis has already been conducted (see Chapters 2 and 4).

The primary instrument of this chapter shall include the methods of systematic star coordinate errata calculation as described in Chapter 5. First of all, we shall demonstrate that seven regions of the Almagest star atlas as described above do actually differ from each other by the system error rate as well as random measurement errata. We shall find errors in ecliptic pole estimation for each of these areas, as well as the values of residual square average star coordinate errata. Moreover, we shall build confidence intervals of systematic error parameters γ_{stat} and φ_{stat} for each of the areas.

Next we shall analyse certain comparatively small celestial areas – constellations and environs of individual stars. The goal of this analysis is to make sure that the discovered values of γ_{stat} and φ_{stat} do in fact possess the nature of systematic errata in substantial parts of the Almagest catalogue, and are by no means a mere result of numerous group errors superimposed over each other and differing from one small group of stars to another.

As a result, we shall calculate the area of the celestial sphere that was measured well enough by Ptolemy. In fact, it turned out rather significant. Our dating of the Almagest shall be based on star coordinates from this very area – one where Ptolemy's calculations were the most precise.

2. SEVEN REGIONS OF THE CELESTIAL SPHERE

2.1. A characteristic of the seven areas that we have discovered in the Almagest atlas

In Chapter 2 we have described seven areas that the celestial sphere can be divided into; they are also very manifest in the Almagest catalogue, qv in fig. 6.1.

In this chapter, we analyse Ptolemy's coordinates of 864 stars in total. These 864 stars were what we rendered the 100 stars of the Almagest to after a filtration of the following sort. Firstly, the so-called *informata* stars were removed due to reasons considered in Chap-

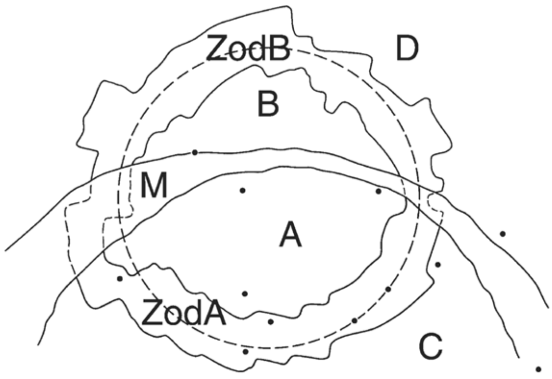

Fig. 6.1. Seven areas that we discovered in the star chart according to the Almagest. Named stars are represented by black dots.

ter 2 – they aren't included in the canonical constellations. Secondly, we have also filtered out the "rejects" and the ambiguously identified stars. Table 6.1 contains precise indications concerning the Almagest stars that a given region includes, and the residual amount of stars after the "filtration" for each area. We have used Bailey's numeration in this table, or star numbers from the Almagest catalogue.

Let us consider fig. 6.1, which represents the division of the celestial sphere into the abovementioned regions. All 12 named Almagest stars are marked as black dots. It is easy to see that the outline of area *A* is very clearly defined by the named stars of the Almagest. One gets the impression that Ptolemy ascribed a special significance to celestial area *A*. This is also confirmed by our preliminary analysis in Chapter 2. As we shall see below, area A turns out the most important for our dating research. It also has to be pointed out that the area in question contains the celestial pole (marked *N*) and the ecliptic pole (marked *P*).

Named stars that surround area *A* must have served Ptolemy as a basis of some sort when he was performing his observations. He referred to them as he moved further towards the centre of area *A*, measuring the coordinates of all the other stars. Measurement errata accumulated as he moved from one star to another. One should therefore expect the stars from region *A* that lay outside the Zodiac to be measured worse in general than zodiacal stars. Half of the Almagest's named stars (6 out of 12) are either part of the Zodiac, or located in its immediate vicinity. The Zodiac includes Regulus, Spica, Antares, Previndemiatrix and Aselli. Procyon is right next to the Zodiac.

Region of the sky in the Almagest	*Bailey's numeration for the region before and after the filtration of the catalogue*	
	before	*after*
A	1-158 and 424-569	249
B	286-423 and 570-711	262
C	847-977	116
D	712-846 and 998-1028	143
M	159-285	94
Zod A	424-569	124
Zod B	362-423 and 570-711	168

Table 6.1. The distribution of the Almagest stars across the celestial areas with the specification of just how many stars remained in each of said areas after the filtration of the catalogue. We were using Bailey's enumeration, or the numbers of the stars as specified in the catalogue of the Almagest.

2.2. The disposition of the ecliptic poles for each of the seven regions of the Almagest star atlas

Let us first locate the disposition of the ecliptic poles for each of the seven celestial regions of the Almagest. In Chapter 5 we demonstrate that the position of the ecliptic pole in relation to the catalogue stars is set by parameters γ_{stat} and φ_{stat}. These parameters are estimated from the catalogue by the application of the minimal square method in accordance with the formulae (5.3.6 and 5.3.7).

Let us calculate the values of parameters γ_{stat} and φ_{stat} for each of the seven celestial regions separately. Afterwards we shall mark each corresponding position of the ecliptic pole in fig. 6.2. In the same illustration we shall also define the motion of the real ecliptic pole $P(t)$ that corresponds to the variations of the alleged dating.

In fig. 6.2 we have used the following segment as an example: it connects the ecliptic pole for celestial area B with the real ecliptic pole for epoch $t = 10$ marked $P(10)$. The length of this segment equals $\gamma^B_{stat}(10)$. The angle between this segment and the line that stands for arc $D(10)\ D'(10)$, whose definition was cited in relation to fig. 5.4 and 5.5; its value is equal to $\varphi^{B}_{stat}(10)$. Obviously enough, any other epoch can be taken as t, ditto area B, and respective values of γ_{stat} and φ_{stat} can be deduced with the aid of fig. 6.2.

Table 6.2 contains the values of $\gamma_{stat}(18)$ and $\varphi_{stat}(18)$ that we have calculated for each of the seven celestial regions. These positions provide an unambiguous definition of the "observer ecliptic pole" for each of the areas. However, we may have just as easily taken any pair of γ_{stat} and φ_{stat} values for a random t. We refer you further to section 5.4. Apart from that, table 6.2 contains the values of $\sigma_{init}(18)$ and the residual σ_{min} square average latitudinal discrepancies resulting from the compensation of the systematic error (see formulae 5.3.2 and 5.3.3). In section 5.4 we demonstrate that σ_{min} does not depend on time moment t under consideration, if we disregard the insubstantial influence of the proper star motion. Therefore, σ_{min} is defined by the ecliptic pole exclusively, which can be estimated statistically for this group of Almagest stars.

As for proper star motion, it has to be pointed out that it hardly affects either the estimated systematic error $\gamma_{stat}(t)$, $\varphi_{stat}(t)$ or the residual square average discrepancy of star coordinates in the Almagest catalogue. Therefore, we can omit all references to the effect of proper motion, although it was obviously always taken into account in our calculations.

We have chosen the value of $t = 18$ for table 6.2 just because this time moment corresponds to the Scaligerian dating of the Almagest.

Further on, Table 6.2 contains the following statistic characteristic of Almagest stellar coordinate precision. The value of $P_{init}(18)$ corresponds to the percentage of the stars whose latitudinal discrepancy doesn't exceed 10' for the dating of 100 A.D. ($t = 18$), 10' being the Almagest catalogue minimal scale step. The value of P_{min} corresponds to the share of the stars whose latitudinal discrepancy doesn't exceed 10' after the compensation of the systematic error. This value is hardly affected by the dating of the observations for large quantities of stars as considered presently.

The disposition of the statistically definable Almagest poles shown in fig. 6.2 as related to the trajectory of the true pole's motion tells us that in every celestial area except C the systematic error of the Almagest catalogue makes the catalogue "more ancient" even as compared to the epoch of Hipparchus. Let us re-

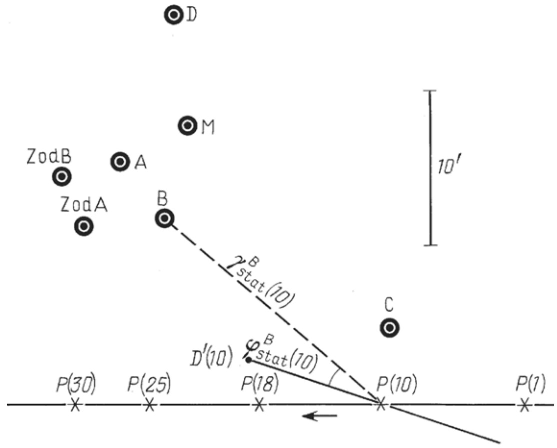

Fig. 6.2. The respective disposition of the mobile ecliptic pole $P(t)$ and the ecliptic poles as estimated for each of the seven parts comprising the Almagest catalogue.

Characteristics	Areas of the Almagest sky						
	A	*B*	*C*	*D*	*M*	*ZodA*	*ZodB*
$\gamma_{stat}(18)$	18.5	13.6	9.7	26.6	19.4	16.4	20.0
$\varphi_{stat}(18)$	34.0	–34.5	–122.5	–52.7	–50.5	–21.7	–23.5
$\sigma_{init}(18)$	20.5	21.8	23.4	27.3	23.0	17.7	24.0
σ_{min}	16.5	19.2	22.5	24.4	20.5	12.8	19.3
$P_{init}(18)$, in %	36.5	35.5	33.6	28.7	37.2	30.6	30.9
P_{min}, in %	50.6	43.5	43.1	35.7	45.7	63.7	44.0

Table 6.2. Calculated values of error parameters $\gamma_{stat}(18)$ and $\varphi_{stat}(18)$ as specified in the Almagest for different celestial regions.

mind the reader that the system error minimum in celestial region C falls over $t \sim 10$, or the year ~900 (900 A.D.). Still, as we have mentioned above, the disposition of the pole of "Ptolemy's Ecliptic" isn't in any way related to the date of the catalogue's compilation. This disposition simply tells us the character and the value of the systematic error made by Ptolemy in the measurements of star coordinates as conducted for one celestial region or another.

Another implication made by fig. 6.2 is that the statistically estimated pole positions for regions *A*, *ZodA* and *ZodB* are rather close to each other – in other words, Ptolemy appears to have made the same systematic error for each of these celestial regions. We shall come back to this fact below, in our analysis of individual Almagest constellations. Furthermore, the ecliptic pole defined by region *B* of the Almagest catalogue is also located next to the pole for groups *A*, *ZodA* and *ZodB*, as we see from fig. 6.2. The position of the pole for area *M* lays further away, and that of area *D* – even further off. Apparently, the systematic error of the Almagest's areas *M* and *D* has a different value than that of area *ZodA*. Area *C* looks like an obvious "reject" in fig. 6.2.

2.3. The calculation of confidence intervals

In the previous section we calculated discrete statistical estimates γ_{stat} and φ_{stat} for the unknown parameters of the Almagest catalogue's systematic error (γ and φ). We have already reminded the reader the definition of confidence intervals in section 5.5. Let us make the visual representation of the result as follows. First we shall build dependence graphs for t and the estimates of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$, where $1 \le t \le 25$. Then we shall draw stripes on the resulting graphs, whose vertical sections shall be the confidence intervals $I_\gamma(\varepsilon)$ and $I_\varphi(\varepsilon)$ with confidence level $\varepsilon = 0.1$. Confidence intervals shall be calculated in accordance with the formulae 5.5.10 and 5.5.11.

The result of these calculations can be seen in figs. 6.3-6.9. More data on the borders of different confidence levels ε and the two values of the alleged Almagest catalogue dating ($t = 7$, or 1200 A.D., and $t = 18$, or 100 A.D.) can be found in table 6.3. This table contains the values of half-widths of confidence intervals $I_\gamma(\varepsilon)$. Let us remind the reader that the centre of the confidence interval for γ and each fixed value of t is the non-shifted estimate of $\gamma_{stat}(t)$, qv in section 5.5.

		1200 A.D.				*100 A.D.*			
Area ↓	ε →	*0.1*	*0.05*	*0.01*	*0.005*	*0.1*	*0.05*	*0.01*	*0.005*
A	x_ε^γ	2.6	3.1	4.1	4.5	2.7	3.2	4.2	4.6
	x_ε^φ	11.7	14.0	18.3	20.0	16.6	19.8	25.9	28.4
B	x_ε^γ	2.7	3.2	4.2	4.6	2.6	3.1	4.0	4.4
	x_ε^φ	14.7	17.4	22.8	25.0	22.1	26.2	34.4	37.6
C	x_ε^γ	4.6	5.5	7.2	7.9	5.1	6.0	7.9	8.7
	x_ε^φ	91.1	108.2	141.9	155.2	60.7	72.2	94.7	103.5
D	x_ε^γ	6.3	7.4	9.8	10.7	7.2	8.6	11.3	12.3
	x_ε^φ	28.3	33.6	44.1	48.2	37.8	44.9	58.9	64.4
M	x_ε^γ	5.4	6.4	8.5	9.2	6.5	7.7	10.1	11.0
	x_ε^φ	28.2	33.5	43.9	48.0	42.4	50.3	66.0	72.2
Zod A	x_ε^γ	2.5	2.9	3.9	4.2	2.5	3.0	4.0	4.3
	x_ε^φ	11.4	13.6	17.8	19.5	18.1	21.5	28.2	30.8
Zod B	x_ε^γ	3.5	4.2	5.5	6.0	3.4	4.1	5.4	5.9
	x_ε^φ	14.3	17.0	22.3	24.4	19.8	23.5	30.8	33.7

Table 6.3. Semi-width values x_ε^γ of confidence interval $I_\gamma(\varepsilon)$ and x_ε^φ of confidence interval $I_\varphi(\varepsilon)$ for different confidence levels of ε and two presumed datings of the Almagest catalogue – 1200 A.D. ($t = 7$) and 100 A.D. ($t = 18$).

The confidence interval $I_\varphi(\varepsilon)$ for φ is, generally speaking, asymmetrical in relation to $\varphi_{stat}(t)$, since this estimate might be shifted. However, the abovementioned asymmetry is insignificant enough, and one may consider $\varphi_{stat}(t)$ the approximate centre of the confidence interval. x_ε^γ stands for the semi-width of interval $I_\gamma(\varepsilon)$, and x_ε^φ – for the semi-width of interval $I_\varphi(\varepsilon)$.

The figures one finds in tables 6.2 and 6.3 imply the following. Almagest area *ZodA* is the most accurately measured celestial region. This is obvious from the fact that the compensation of the discovered systematic error for this group of stars allows reducing the square average error to 12.8'. Also, it turned out that 64% of the stars ended up with a latitudinal discrepancy of less than 10'.

The second most precise group of stars pertains to the Almagest area *A*, where the square average latitudinal discrepancy became reduced to 16.5' after the compensation of the systematic error. The share of stars whose latitudinal discrepancy is under 10' has grown to over 50% in this area.

Confidence intervals $I_\gamma(\varepsilon)$ and $I_\varphi(\varepsilon)$ for celestial areas *ZodA* and *A* turned out to be of similar sizes, qv in table 6.3, although the precision of measurements is higher in area *ZodA*. This is explained by the heterogeneous quantities of stars for these parts. The less stars, the greater the size of the confidence interval; the latter is reduced by higher measurement precision.

The data from Table 6.2 confirm Ptolemy's claimed precision of 10', insofar as stellar latitudes are concerned, at least.

The next best measured groups of Almagest stars are concentrated in areas *B* and *ZodB*. Their precision characteristics are rather close to each other. The residual square average error is approximately equal to 19'. Stars with a latitudinal discrepancy of under 10' constitute 44% of these groups. The positions of the ecliptic pole calculated by these Almagest sky parts seem close to the pole positions of areas *A* and *Zod A* at a cursory glance; however, they end up in respective confidence intervals only with sufficiently small values of $\varepsilon \approx 0.01$, which means that the systematic errata of celestial areas *B* and *ZodB* may differ from those of *A* and *ZodA*. Moreover, the stars in areas *A* and *ZodA* were measured with substantially greater precision than those in areas *B* and *ZodB*. Below we shall cite more evidence that testifies to this.

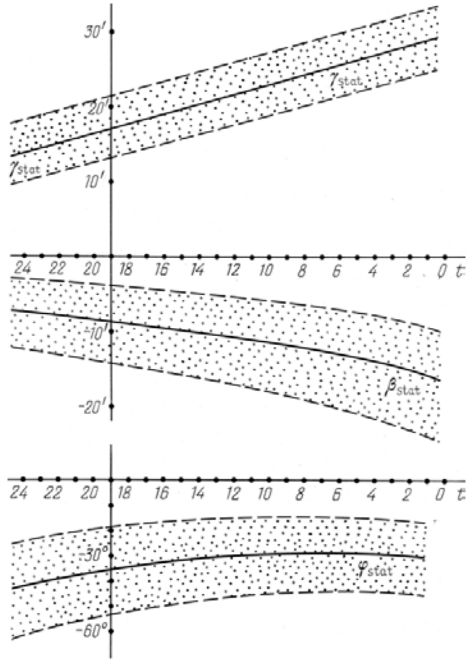

Fig. 6.3. The behaviour of systematic errors $\gamma_{stat}(t)$, $\varphi_{stat}(t)$ and $\beta_{stat}(t)$ for celestial region *A* in the Almagest.

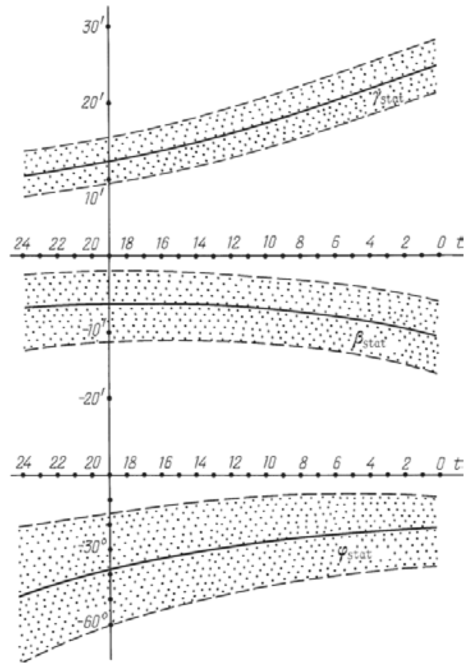

Fig. 6.4. The behaviour of systematic errors $\gamma_{stat}(t)$, $\varphi_{stat}(t)$ and $\beta_{stat}(t)$ for celestial region *B* in the Almagest.

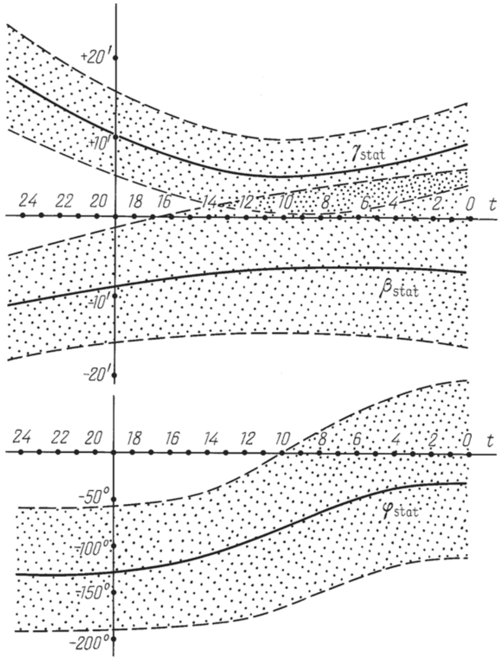

Fig. 6.5. The behaviour of systematic errors $\gamma_{stat}(t)$, $\varphi_{stat}(t)$ and $\beta_{stat}(t)$ for celestial region *C* in the Almagest.

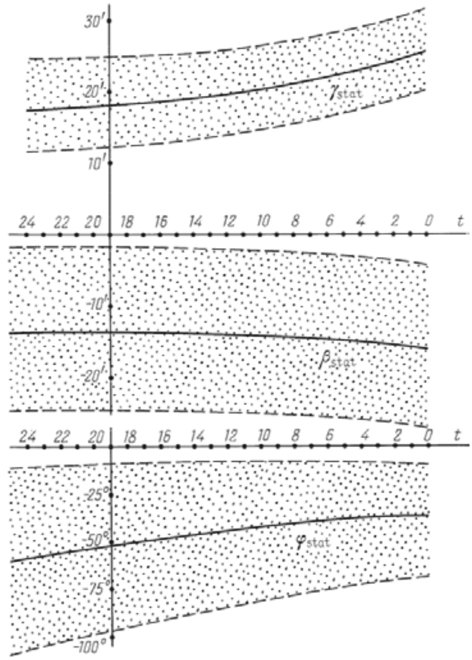

Fig. 6.7. The behaviour of systematic errors $\gamma_{stat}(t)$, $\varphi_{stat}(t)$ and $\beta_{stat}(t)$ for celestial region *M* in the Almagest.

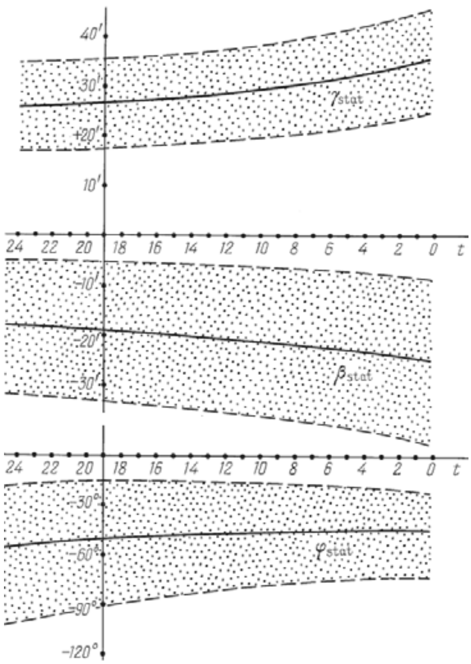

Fig. 6.6. The behaviour of systematic errors $\gamma_{stat}(t)$, $\varphi_{stat}(t)$ and $\beta_{stat}(t)$ for celestial region *D* in the Almagest.

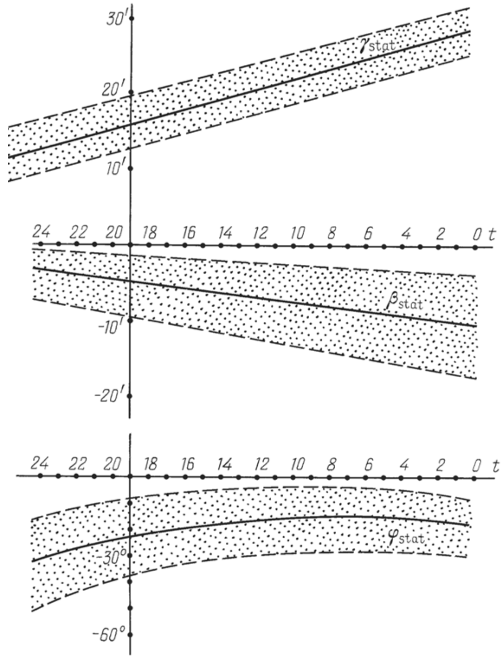

Fig. 6.8. The behaviour of systematic errors $\gamma_{stat}(t)$, $\varphi_{stat}(t)$ and $\beta_{stat}(t)$ for celestial region *ZodA* in the Almagest.

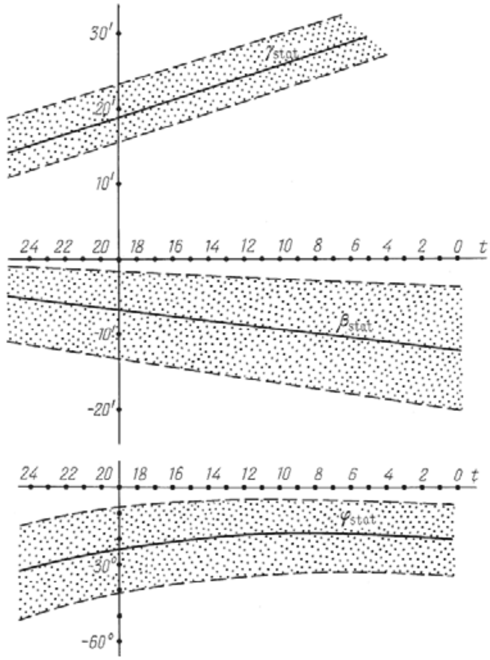

Fig. 6.9. The behaviour of systematic errors $\gamma_{stat}(t)$, $\varphi_{stat}(t)$ and $\beta_{stat}(t)$ for celestial region *ZodB* in the Almagest.

Parameters		*Celestial region in the Almagest atlas*	
		ZodA	*A*
a_{11}		1.11	0.82
a_{12}		0.042	–0.03
a_{22}		0.073	0.13
σ_{min}		12.8'	16.5'
$\Delta\sigma$	$\varepsilon = 0.1$	1.3'	1.2'
	$\varepsilon = 0.05$	1.8'	1.7'
	$\varepsilon = 0.01$	3.0'	1.8'
	$\varepsilon = 0.005$	3.5'	3.3'
σ_{max}	$\varepsilon = 0.1$	14.1'	17.7'
	$\varepsilon = 0.05$	14.6'	18.2'
	$\varepsilon = 0.01$	15.8'	19.3'
	$\varepsilon = 0.005$	16.3'	19.8'

Table 6.4. The values of $a(11)$, $a(12)$ and $a(22)$ as calculated for the Almagest, assuming the date of its compilation to be close to 100 A.D. ($t = 18$).

The stars in areas *C*, *D* and *M* were measured worse than those in areas *A* and *B*. Moreover, the values of γ_{stat} and φ_{stat} estimates only end up inside confidence intervals of areas *A*, *ZodA*, *B* and *ZodB* when the values of ε are very small indeed, which means that we must allow for the existence of such systematic errata in areas *C*, *D* and *M* that differ from the

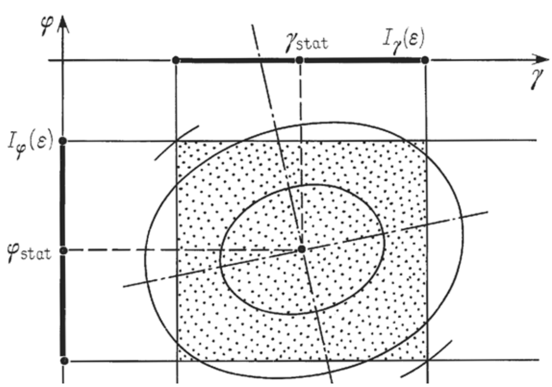

Fig. 6.10. Estimating the allowable variations of the square average latitudinal discrepancy values.

systematic errors pertinent to celestial regions *A*, *ZodA*, *B* and *ZodB*.

The analysis of tables 6.2 and 6.3 has already made us enquire about the values of the square average error that one should consider great and small. Let us refer to the sensitivity analysis as described in Chapter 5. The solution scheme can be seen in fig. 6.10.

Let us draw the ellipsoidal level curves of function $\sigma^2(\gamma, \varphi, t)$ on coordinate plane (γ, φ) according to formula 5.3.9. We shall draw the rectangle $R(\varepsilon)$ on the same plane, with coordinate projections $I_\gamma(\varepsilon)$ and $I_\varphi(\varepsilon)$. In fig. 6.10 it is the shaded rectangle. In this case, the probability that the true value of system error (γ, φ) lays inside this rectangle is $1 - 2\varepsilon$ or greater. Let us find $\sigma^2_{max}(\varepsilon) = max\,\sigma^2(\gamma, \varphi, t)$, where the maximum is taken for each of the pairs $(\gamma, \varphi) \in R(\varepsilon)$. The resulting value of $\sigma_{max}(\varepsilon)$ defines the permissible square average discrepancy with a confidence level of $1-2\varepsilon$, whereas the difference of $\sigma_{max}(\varepsilon) - \sigma_{min}$ defines the permissible expansion of the square average discrepancy due to the lack of sufficient precision in the estimation of parameters γ and φ by the values of γ_{stat} and φ_{stat}.

Table 6.4 contains the values of a_{11}, a_{12}, a_{22} for celestial areas *A* and *ZodA* for the time moment of $t = 18$; they define the level curves of the square average error. These level curves are calculated with the aid of formula 5.3, which stipulates the measurement of γ in arc minutes and φ in degrees. The table also contains the values of $\Delta\sigma = \sigma_{max}(\varepsilon) - \sigma_{min}$ calculated for the "extreme" values of $\varepsilon = 0.1$ and $\varepsilon = 0.005$. It has to be said that the resulting values appear to change little over time. These figures demonstrate the obvious precision division between areas *A* and *ZodA* on the one hand, and *B* and *ZodB* on the other. Indeed, even with the confidence level of $1 - 2\varepsilon = 0.99$, the square average error value of the confidence area constructed for the region *ZodA* remains less than the minimal error value of celestial regions *B* and *ZodB*.

A similar statement shall also be true for celestial region *A*. Although σ^A_{max} of region *A* is greater than σ^B_{max}, this is only true for $\varepsilon \leq 0.01$. Other values make error levels of celestial regions *A* and *B* substantially different, or separated by a statistical criterion. It must be added that the stars in the *ZodA* group are just as different from their counterparts from group *A* precision-wise, since for all ε values considered the value of σ_{max} found for *ZodA* is less than σ_{min} calculated for region *A*.

Furthermore, table 6.3 demonstrates that the parameter φ_{stat} cannot be calculated with sufficient precision, especially for the "poor quality" regions *C, D* and *M*. This is confirmed by the sizes of confidence intervals $I_\varphi(\varepsilon)$. For example, the full range of this interval exceeds 180 degrees in case of area *C*.

3. OUR ANALYSIS OF INDIVIDUAL ALMAGEST CONSTELLATIONS

3.1. The compiler of the Almagest may have made a different error in case of every minor constellation group

Further analysis is necessary due to the following problem. Parameters γ_{stat} and φ_{stat}, which define the systematic error, have been found for some large group of stars. They correspond to the turn of the ecliptic that minimises the square average discrepancy for the stars contained in this group. However, one must not a priori exclude the possibility that the compiler made a separate group error in case of every small star group such as an individual constellation. In this case, parameters γ_{stat} and φ_{stat} are but average meanings of the true group errata, and will be of little use to us for this reason.

We have to note that the sizes of confidence intervals for the values of φ_{stat} found in Section 2 are rather substantial. This may be explained by the low sensitivity of latitudinal discrepancies to the turn angle φ as well as the "non-systematic" nature of the φ_{stat} error. In other words, it is possible that parameters γ_{stat} and φ_{stat} have a different nature, namely, γ_{stat} is the result of an observer's error that affects all stars (an error in the estimation of the ecliptic's position), whereas φ_{stat} is the averaged value of numerous individual errors. Such a difference in the behaviour of the parameters is easy to explain if we consider the primary astronomical instrument of the Almagest epoch, for instance – the armillary sphere (see Chapter 1). The angle between the equatorial and ecliptic plane is fixed once and forever in the very construction of this instrument. If there was an defect in the latter, it would affect the coordinates of each and every star measured with the aid of this armillary sphere. The error in the estimated value of angle φ is of an altogether different nature. It is individual for each star and changes as the observer measures the coordinates of several consecutive stars.

One must therefore find the group errors characteristic for individual Almagest constellations and compare them to the systematic error of *ZodA*, the best measured group of Almagest stars.

3.2. The calculation of systematic errors for individual groups of constellations in the Almagest

The present section analyses a total of 21 small groups of Almagest stars. Their list can be found in Table 6.5, whose structure is completely identical to that of Table 6.1. Our only additional indication concerns the principle of selecting limited stellar configurations. All of the above are zodiacal constellations from the Almagest, likewise the environs of named stars, with the exception of Canopus and Previndemiatrix (made for abovementioned reasons), as well as Procyon, due to the paucity of stars in its environment.

Almagest star group	*Bailey's star numbers for the group*	*Number of stars in a group*
1. Zodiacal constellations		
Aries	362-371, 373, 374	12
Taurus	380-388, 390, 391, 393-410	29
Gemini	424-440	17
Cancer	449-454	6
Leo	462-481, 483-488	26
Virgo	497-516, 518-520	23
Libra	529-534	6
Scorpio	546-565	20
Sagittarius	570-573, 575-583, 585, 586, 590, 591, 593, 594, 596-598	22
Capricorn	601-608, 610-627	26
Aquarius	629-650, 652-656, 658-660, 662-668	37
Pisces	674-695, 697, 699-701, 704-706	29
2. Environs of named Almagest stars		
Antares	546-569	24
Cappella	220-233	14
Aquila	286-300	15
Vega = Lyra	149-158	10
Arcturus	88-96, 98, 100-110	21
Sirius	812, 818-835, 837-846	29
Spica	497-503, 505-515, 518-526	27
Regulus	462-481, 483-488, 491-493	29

Table 6.5. Stellar compound of 21 Almagest star groups; for each of the latter, the values of systematic (group) errors were calculated. These groups include all the zodiacal constellations of the Almagest, as well as the neighbourhood of 12 named Almagest stars, with the exception of Canopus and Previndemiatrix. The table contains Bailey's enumeration, or star numbers as given in the Almagest catalogue.

The location of group errors for individual Almagest constellations is associated with the following problems. Let us consider a certain star group G and find the corresponding values of γ^G_{stat} and φ^G_{stat} by applying the method of minimal squares. This will also define the minimal possible residual square average discrepancy σ^G_{min}, as well as the share of stars whose residual latitudinal discrepancy is less than 10'. This will also define P^G_{min} in relation to the time moment $t = 18$. However, due to the small sizes of certain star groups, the statistical discrepancy of estimates γ^G_{stat} and φ^G_{stat} is too great to serve as a basis for justified corollaries.

However, the value of σ^G_{min} defines the lower boundary of possible square average errata for group G. This minimal value of possible error results from turning the coordinate system by angles γ^G_{stat} and φ^G_{stat}. Obviously enough, the values of γ^G_{stat} and φ^G_{stat} can greatly differ from those of γ_{stat} and φ_{stat}, which were calculated for a larger number of stars that had included group G.

The identity criterion of group error for group G and the systematic error calculated for a large number of stars could be expressed as the approximated equation $\sigma^G_{min} \approx \sigma^G_1$, where σ^G_1 is the residual square average discrepancy for group *G* after the coordinate system is rotated by angles γ_{stat} and φ_{stat}. Indeed, the above approximated equation means that γ_{stat} and φ_{stat} are "almost" optimal values. In order to support this criterion, let us define the auxiliary values of P^G_{min} and P^G_1, which stand for the share of stars from group *G* whose latitudinal discrepancy does not exceed 10' after the respective rotations of (γ^G_{stat} and φ^G_{stat}) and (γ_{stat} and φ_{stat}). Should we also observe a case of $P^G_{min} \approx P^G_1$, we can conclude that group *G* does indeed possess the same systematic error value as the stars of a greater group. We must note that the latter approximate proportion is not implied by the former, but happens to prove our claim independently. It also needs to be pointed out that both proportions are temporally independent, if we are to disregard the proper star motion. Therefore, their practical verification can only be conducted for a single moment in time – any one such moment, that is.

We have calculated the values of σ^G_1 and P^G_1 for different Almagest groups *G* and the time moment of $t = 18$. Let us reiterate that these values equal to the respective square average latitudinal discrepancy and the share of stars whose latitudinal discrepancy value does not exceed 10', given that the pole of the ecliptic coincides with the pole defined for the most accu-

Star group	*Indication of G*	σ^G_{init}	σ^G_{min}	σ^G_1	P^G_{init}	P^G_{min}	P^G_1
1. Zodiacal constellations							
Aries	Z1	19.7	17.2	18.9	45.5	45.5	72.7
Taurus	Z2	23.2	18.1	20.6	27.6	41.4	41.4
Gemini	Z3	17.8	10.5	11.0	29.4	82.4	58.8
Cancer	Z4	13.8	4.3	5.2	33.3	100.0	100.0
Leo	Z5	20.2	11.1	11.2	19.2	65.4	65.4
Virgo	Z6	18.4	13.6	14.4	39.1	56.5	47.8
Libra	Z7	8.4	6.1	9.3	83.3	83.3	83.3
Scorpio	Z8	18.8	13.7	15.1	30.0	65.0	55.0
Sagittarius	Z9	16.4	14.3	15.8	30.4	60.9	60.9
Capricorn	Z10	16.2	10.6	11.3	42.3	65.4	57.7
Aquarius	Z11	28.6	17.3	19.2	18.4	44.7	44.7
Pisces	Z12	22.5	21.5	21.7	51.7	41.4	34.5
2. Environs of named Almagest stars							
Antares	S1	17.7	12.6	13.8	33.3	70.8	58.3
Acelli	S2	15.7	11.0	12.1	33.3	58.3	66.7
Cappella	S3	34.6	30.3	34.0	35.7	14.3	64.3
Aquila	S4	24.0	23.7	26.7	40.0	33.3	13.3
Vega = Lyra	S5	20.0	14.1	17.1	50.0	60.0	30.0
Arcturus	S6	24.2	17.2	20.0	19.0	38.1	28.5
Sirius	S7	15.2	11.9	25.9	47.4	52.6	15.8
Spica	S8	17.9	14.1	14.5	44.4	48.1	48.1
Regulus	S9	25.2	21.0	21.1	17.2	58.6	58.6

Table 6.6. Calculation results for the 21 Almagest star groups. Here σ^G_{init}, σ^G_{min}, σ^G_1 correspond to square average latitudinal discrepancies in group *G* – the initial and the remaining, as well the one that we come up with after compensating the systematic error in *G* as estimated for *ZodA*. We also cite the stellar percentage values of P^G_{init}, P^G_{min}, P^G_1 with a minimal latitudinal discrepancy of 10'.

rately measured group of stars in area *ZodA*. In other words, the condition is that the group errors must equal the values of γ_{stat}^{ZodA} and φ_{stat}^{ZodA}.

The square average latitudinal discrepancy and the percentage of stars whose latitudinal discrepancy value doesn't exceed 10' (in group *G*, without the compensation of the systematic error) were transcribed for $t = 18$ as σ_{init}^{G} and P_{init}^{G}, respectively.

If the value of σ_1^G exceeds the minimal possible value of σ_{min}^{G}, but very slightly so, we are entitled to assume that the group error value of star group *G* equals the systematic error value of celestial region *ZodA*. The difference between the values of P_1^G and P_{min}^{G} is yet another proximity criterion for group error and systematic error. Let us remind the reader that the values σ_{min}^{G} and σ_1^G are temporally independent for the immobile stars and only marginally depend on time in case of their mobile counterparts. A similar statement shall be true for the stars that end up within the 10' interval of the latitudinal discrepancy.

Table 6.6 contains the numeric data that we have calculated. A more visual representation thereof can be found in figs. 6.11 and 6.12. Fig. 6.11 contains the information about the values of σ_{min}^{G} and σ_1^G, as well as P_1^G and P_{min}^{G}, for all the zodiacal constellations of the Almagest (indicated Z1, … , Z12). Fig. 6.12 contains respective results for the environs of the named Almagest stars; they are marked S1, … , S9. One must say that the environs of the named Zodiacal stars in the Almagest do not fully correspond with the respective Zodiac constellation. These environs are constituted by a group of stars from this constellation, which have received a name in Bayer's system. These stars are usually the brightest and the most reliably identifiable stars of the Almagest, which makes them more solid corollary basis.

3.3. Group errors for individual constellations from the well measured celestial region of the Almagest are virtually identical to the systematic error discovered as a characteristic of this area in general

The key implication of the cited graphs and of Table 6.6 is that the zodiacal constellations from celestial region *ZodA* (namely, Gemini, Cancer, Leo, Virgo, Libra and Scorpio) possess the following remarkable quality in the Almagest. The square average error σ_1 and the percentage of stars with the maximal latitudinal discrepancy of 10' calculated under the assumption that the group error is equal to (γ_{stat}^{ZodA}, φ_{stat}^{ZodA}) are only marginally different from the values of σ_{min} and P_{min} calculated for the optimal ecliptic pole position in the constellation under study. The greatest discrepancy between the two was noted in the "most orderly" constellation of Libra, where no value of σ_{init}, σ_{min} or σ_1 exceeds 10', and $P_{init} = P_{min} = P_1 =$ 83,3%. Such is the percentage of stars whose latitudinal discrepancy value is less than 10'. The equation $P_{init} = P_{min} = P_1$ is easy to explain – the constellation in question all but lays on the equinoctial axis, thus remaining quite unaffected by the turn.

However, this corollary may also be true for the constellations from celestial region *ZodB*, although with more details to take into account. However, the veracity or inveracity of this corollary is of no importance to us presently, since celestial region *ZodB* doesn't contain any named Almagest stars.

We must nevertheless point out a single curious fact that was found out in relation to the constellation of Aries. Although the value of σ_1 became lower in comparison to σ_{init} after the compensation of the systematic error discovered earlier (one must also note that the difference between σ_{min} and σ_{init} is rather small), but $P_1 >> P_{init} = P_{min}$ – in other words, the shift of the ecliptic pole into the position calculated for *ZodA* made it possible to raise the share of well-measured Almagest stars in the constellation of Aries to 72.7%.

The general conclusion resulting from our the consideration of all zodiacal constellations is as follows. If the proportion $\sigma_{min} << \sigma_{init}$ is true for the optimal value of σ_{min}, the conjecture that the group error equals the systematic error for celestial region *ZodA* and the ensuing compensation of this error lead us to the proportion of $\sigma_1 << \sigma_{init}$; other valid proportions include $P_1 >> P_{init}$ and $P_{min} >> P_{init}$. This is true of the following Almagest constellations: Gemini, Cancer, Leo, Virgo, Scorpio, Capricorn and Aquarius.

If the value of σ_{min} is close to σ_{init}, $\sigma_{min} \le \sigma_1 \le \sigma_{init}$ as a rule, and the effect of placing the ecliptic pole into the position that corresponds to area *ZodA* is hardly manifest at all. This is true of the Aries constellation

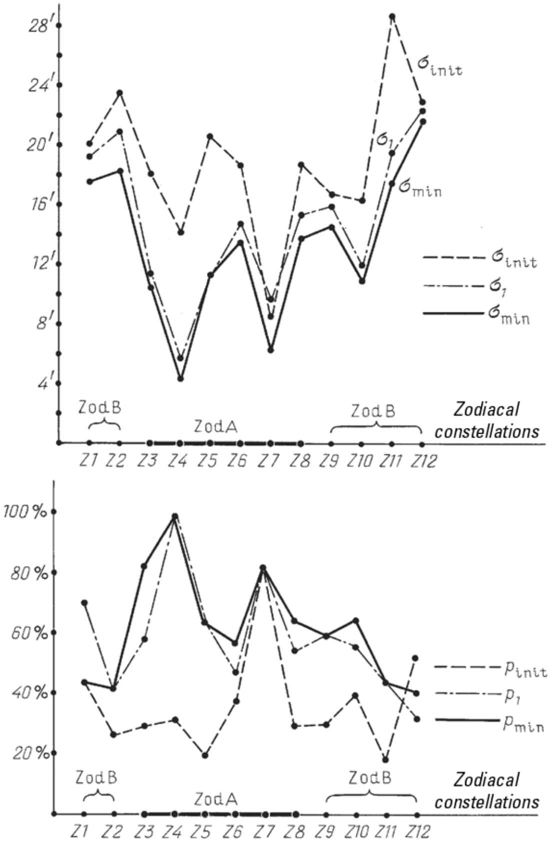

Fig. 6.11. The dependencies of σ_{min}, σ_1, σ_{init}, P_{min}, P_1, P_{init} for the zodiacal constellations.

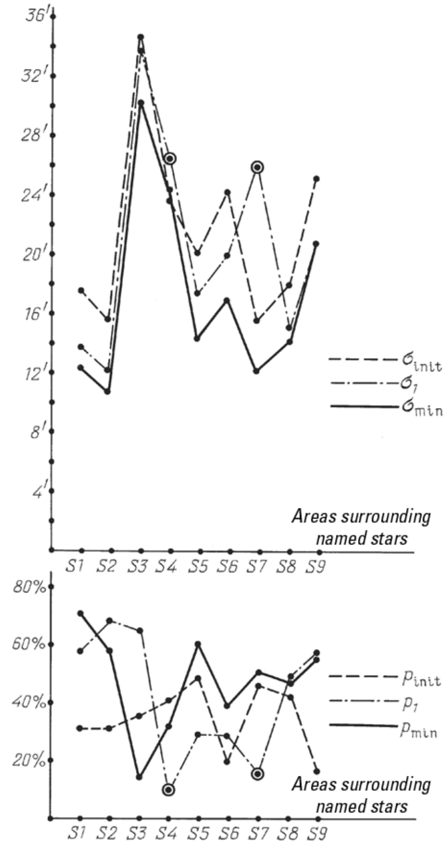

Fig. 6.12. The dependencies of σ_{min}, σ_1, σ_{init}, P_{min}, P_1, P_{init} for the areas around named stars.

(as we have pointed out, the percentage of well-measured stars grew dramatically in case of Aries), as well as Taurus, Libra, Sagittarius and Pisces.

Out of the constellations pointed out above, good precision characteristics of Libra from celestial area *ZodA* remain virtually unchanged after the shift of the ecliptic pole from the optimal position to the position that corresponds to *ZodA*. Precision characteristics of Aries become even better after this operation, and those of all the other constellations remain average.

Taurus is a typical example, with $\sigma_{init} = 23.2'$, $\sigma_{min} = 18.1'$, $\sigma_1 = 20.6'$, $P_{init} = 27.6\%$ and $P_{min} = P_1 = 41.4\%$. The constellation of Pisces differs from all the other Almagest constellations, with $P_{min} < P_{init}$ and $P_1 < P_{init}$, given that $\sigma_{init} \approx \sigma_{min} \approx \sigma_1$.

3.4. How the compensation of the systematic error that we have discovered affects the precision characteristics of the environs of named stars

The situation with the environs of named stars in the Almagest is more diverse. First of all, let us point out the environs of Aquila and Sirius. In both cases, the compensation of the discovered systematic error, characteristic for celestial region *ZodA*, leads to the following. Firstly, we observe a growth of the square average latitudinal discrepancy, which is rather substantial in case of Sirius – from 15.2' to 25.9'. Secondly, the share of well measured stars shrinks (from 40% to 13.3% for Aquila, and from 47.4% to 15.8% for Sirius). The obvious conclusion to make is that the group error of the compiler made during the measurements of the environs of Aquila and Sirius is substantially different from the systematic error of celestial region *ZodA*. Unfortunately, it is impossible to calculate these errors veraciously. Therefore, Sirius and Aquila were excluded from further consideration.

The environs of other named stars have basically the same properties as the zodiacal constellations – namely, stars from the environs of Antares, Acelli, Arcturus, Spica and Regulus greatly reduce the square average error, bringing it close to the minimal possible values after the compensation of the group error, which equals the systematic error for region *ZodA*. The percentage of stars whose latitudinal discrepancy value is smaller than 10' (P_1) shall dramatically grow as compared to the initial value of P_{init}. The environs of Cappella have the same property as the constellation of Aries – namely, the square average latitudinal discrepancy of this area doesn't change much after the shift of the ecliptic pole from the initial position to the optimal position and then also into the position calculated for celestial region *ZodA*. However, in the last of said positions the share of stares that fit into the ten-minute latitudinal discrepancy value grew drastically in the vicinity of Cappella, reaching 64.3%. For comparison, let us point out that in the initial position this share equalled 35.7%, and as little as 14.3% in the optimal position dictated by the square average latitudinal discrepancy. On the contrary, the stars neighbouring with Vega demonstrated a substantial reduction of the square average latitudinal discrepancy. However, when we shifted the ecliptic pole into the position characteristic for celestial region *ZodA*, the number of stars with the latitudinal discrepancy value of 10 minutes and less was reduced substantially. Therefore, the nature of group errors in the environs of Vega and Cappella remains unclear. Little wonder – one might as well recollect that these stars lay at quite some distance from the celestial region of *ZodA*.

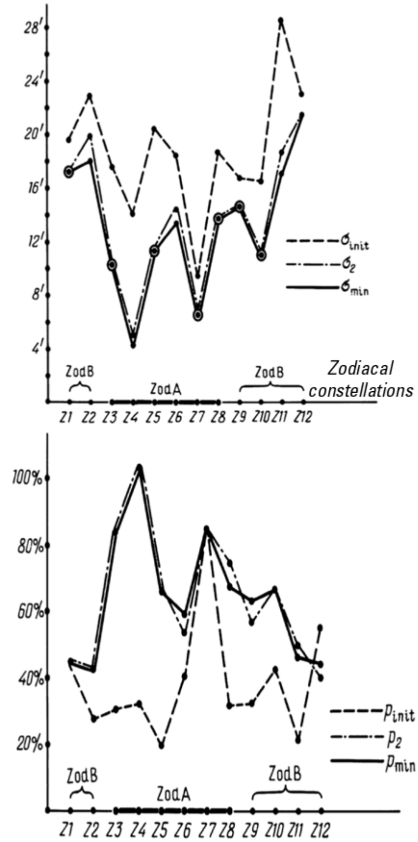

Fig. 6.13. The dependencies of σ_{min}, σ_2, σ_{init}, P_{min}, P_2, P_{init} for the zodiacal constellations.

3.5. The discovery of a single systematic error made by the compiler of the Almagest for the region of *ZodA* and the majority of named stars

Although we have discovered a certain proximity between the characteristics of σ_1 and P_1, respectively to σ_{min} and P_{min} (which testifies to the systematic nature of γ_{stat}), the issue of whether or not the error of φ_{stat} might be systematic as well remains open. Let us solve it in the following manner. Let us consider some individual Almagest constellation. We shall not go beyond the zodiacal constellations – the six named stars pertain to the Zodiac, at any rate. Let us calculate the characteristics of σ_2 and P_2 for these constellations, which can be done as follows. The first characteristic is the residual square average discrepancy, and the second – the share of stars in a constellation whose latitudinal discrepancy does not exceed 10'. Both characteristics result from the statistical error γ_{stat}^{ZodA}, calculated for region *ZodA*, and $\varphi^{(2)}$, calculated as a necessary pre-requisite for the minimization of the σ_2 error. In other words, this is what we come up with for constellation *G*:

$$\sigma_2^G = \sigma_2^G(t) = \min_{\varphi} \sigma^G(t) = \min_{\varphi} \sigma^G(\gamma_{stat}^{ZodA}, \varphi, t),$$

$$\varphi^{(2)} = \arg\min_{\varphi} \sigma^G(\gamma_{stat}^{ZodA}, \varphi, t).$$

Let us compile table 6.7, which is similar to table 6.6. Moreover, some data recur for better demonstrability. In table 6.7 the values of σ_1 and P_1 are replaced by σ_2 and P_2. Let us also draw these data as fig. 6.13, which is similar to fig. 6.11. Both the table and the drawing make it obvious that the compensation of systematic error γ_{stat}^{ZodA} in zodiacal constellations from celestial area *ZodA* and the variation of the φ value may give us minimal possible values of σ_2, which are very close to σ_{min} or even equal to σ_{min}. Likewise, the value of P_2 will be close to P_{min} or equal thereto. Remarkably enough, the same is true for the constellations from celestial region *ZodB*.

All of the above proves it beyond any doubt that the value of γ_{stat}^{ZodA} that we have discovered is indeed the systematic error made by the compiler of the Almagest catalogue as he measured the stars from celestial region *ZodA*, as well as named stars, with the exception of Sirius, Aquila and Canopus. The value of φ_{stat}^{ZodA} can be an averaged result of many individual measurement errors, and there is no reason to consider it a systematic error. Moreover, the value of φ_{stat} is calculated rather roughly, which makes it rather uninformative in this respect.

4. COROLLARIES

Corollary 1. It has been proven statistically that the ecliptic poles of stars from celestial regions *A* and *ZodA* are very close to one another, which makes the values of the systematic error made by the compiler of the Almagest in these parts of the sky the same.

Corollary 2. The statistical analysis that we have conducted gives one no reason to believe that the systematic error values of the Almagest catalogue for celestial regions *C, D, M, B* and *ZodB* have anything in common with such values characteristic for areas *A* and *ZodA*. Systematic errors of areas *C, D,* and *M* are very likely to differ from their counterparts in areas *A* and *ZodA*. We can say nothing of any substance about the errors that characterise celestial regions *B* and *ZodB* in the Almagest, since the numer-

Star group	*Indication of G*	σ_{init}^G	σ_{min}^G	σ_2^G	P_{init}^G	P_{min}^G	P_2^G
Zodiacal constellations							
Aries	Z1	19.7	17.2	17.2	45.5	45.5	45.5
Taurus	Z2	23.2	18.1	20.2	27.6	41.4	41.4
Gemini	Z3	17.8	10.5	10.6	29.4	82.4	82.4
Cancer	Z4	13.8	4.3	4.5	33.3	100.0	100.0
Leo	Z5	20.2	11.1	11.1	19.2	65.4	65.4
Virgo	Z6	18.4	13.6	14.4	39.1	56.5	52.2
Libra	Z7	8.4	6.1	6.1	83.3	83.3	83.3
Scorpio	Z8	18.8	13.7	13.7	30.0	65.0	70.0
Sagittarius	Z9	16.4	14.3	14.4	30.4	60.9	56.5
Capricorn	Z10	16.2	10.6	10.6	42.3	65.4	65.4
Aquarius	Z11	28.6	17.3	18.7	18.4	44.7	47.4
Pisces	Z12	22.5	21.5	21.7	51.7	41.4	37.9

Table 6.7. Calculation result for the zodiacal constellations of the Almagest. Here σ_{init}^G, σ_{min}^G, σ_2^G represent the square average latitudinal discrepancies in group *G* – the initial and the remaining, as well the one that we come up with after compensating the systematic error in *G* as estimated for *ZodA* with the optimal choice of parameter φ. We also cite the stellar percentage values of P_{init}^G, P_{min}^G, P_2^G, as calculated after similar compensation, with a minimal latitudinal discrepancy of 10'.

ical material that we have at our disposal doesn't permit anything in the way of an unambiguous statistical conclusion.

Corollary 3. The precision of star coordinate measurements is much higher for *A* and *ZodA* than it is in case of any other celestial region.

Corollary 4. The residual square average latitudinal discrepancy for celestial region *ZodA* equals 12.8' in the Almagest. About 2/3 of all stars from this part of the sky have the latitudinal discrepancy of less than 10', which make them fit the declared 10' precision margin of the Almagest catalogue. Corresponding values for celestial region *A* equal 16.5' and 1/2.

Corollary 5. A study of the Zodiacal constellations and the environs of named stars in the Almagest makes it possible to conclude that parameter γ, which stands for the error in the angle of the ecliptic, is a systematic error. As for parameter φ, it may well be a squared value of group or individual errors.

Corollary 6. Group error γ for the constellations of Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius and Capricorn, as well as the environs of Antares, Acelli, Arcturus, Spica and Regulus, happens to be close to the systematic error of γ_{stat}^{ZodA}, which is characteristic for *ZodA*, the part of the sky measured best in the Almagest, and might even coincide therewith.

Corollary 7. Nothing definite can be said about the values of group errors made by the compiler of the Almagest in cases of Aries and Taurus. They may coincide with the errata discovered for *ZodA* or be different from their values. The errata in the environs of the named stars Cappella and Vega cannot be calculated, either.

Corollary 8. Group errors in the environs of Sirius and Aquila differ from the error that is characteristic for celestial region *ZodA*. However, we haven't managed to calculate the values of these errors. The group error made for the constellation of Pisces is also likely to differ from γ_{stat}^{ZodA}.