CHAPTER 9

The application of our method to the dating of other mediaeval catalogues

1. INTRODUCTION

Above we have described the methods of statistical analysis and dating of stellar catalogues and applied it to the Almagest star catalogue. It would be of interest to apply the very same method to the dating of other catalogues compiled with the aid of instruments similar to Ptolemy's, or naked eye observations.

The present chapter contains a study of the star catalogues compiled by Ulugbek, Al-Sufi, Tycho Brache and Hevelius. The catalogue of Al-Sufi turned out to be a mere clone of the Almagest. However, several observers already pointed this out – see [1339], [1119] and [1120], for instance. We are apparently the first to have conducted an in-depth statistical analysis of stellar latitude errors in the catalogues of Ulugbek, Tycho Brahe and Hevelius. The precision of these catalogues turned out a great deal worse than it was believed, qv below. The discrepancy is the greatest for the catalogue of Hevelius – a factor of 100x or 200x, no less.

We first dated the catalogues of Tycho Brahe and Ulugbek. The dating of Tycho Brahe's observations is presumed to be well-known – 1570-1600 A.D. Our method yields a dating of Tycho Brahe's catalogue that concurs with this period quite well.

In case of Ulugbek's catalogue, the possible interval that we calculated also covers the Scaligerian dating of its compilation, namely, 1437 A.D. However, this interval also intersects with the above possible dating interval as calculated for the Almagest catalogue. What we should point out in this respect is that the precision of both Ptolemy's catalogue and Ulugbek's is virtually the same; therefore, it is possible that their catalogues were indeed compiled around the same time.

2. TYCHO BRAHE'S CATALOGUE

2.1. A general characteristic of Tycho Brahe's catalogue and the result of our dating

The edition of Tycho Brahe's catalogue that we chose for research had originally been Kepler's and dates to 1628; it was subsequently reprinted in [1024]. Tycho Brahe's catalogue is rendered to the epoch of 1600 A.D. by longitudinal precession in this edition. The structure of the catalogue coincides with that of the Almagest as well as the order in which the constellations are listed – with the exception of several constellations from the very end of the Almagest catalogue which aren't present in the work of Tycho Brahe. There are 1005 stars altogether in Tycho

Brahe's catalogue. The construction principle of the instruments used by Tycho Brahe is the same as of those described by Ptolemy. Therefore, despite the numerous improvements, and the highly elolved instrument manufacture procedure, Tycho Brahe's level of precision is comparable to that of the Almagest catalogue, albeit somewhat better. It equals 2'-3' as opposed to the 10'-15' of the Almagest. The drastic leap in astronomical observations appears to have taken place somewhat later, after the invention of the telescope.

The dating of Tycho Brahe's observations is assumed to be known very well – namely, 1570-1600. Dating Tycho's catalogue independently from consensual chronology, using no other data but the stellar coordinates contained in the catalogue, gives us an opportunity of testing the dating method that we suggest using the example of a problem whose solution is known a priori. The resultant dating interval is as follows: 1510-1620 A.D. It has a length of 110 years and covers the time interval of Tycho Brahe's observations. Let us point out that the length of this interval is some 6 times less than what we got for the Almagest (roughly 700 years) using the same method. The reason is that Tycho Brahe's observation precision level is about 5-6 times higher than Ptolemy's.

2.2. The analysis of Tycho Brahe's latitudinal errors and the removal of the "rejects"

In our dating of Tycho Brahe's catalogue we have once again used nothing but stellar latitudes, the reasons being the same as in case of the Almagest. The identifications of Tycho Brahe's catalogue stars on the modern celestial sphere were taken from Bailey's work ([1024]).

It is assumed that Tycho Brahe may have observed only about 800 of the 1005 stars included in his catalogue ([65], page 126). If this is indeed so, the data contained in his catalogue are not of a homogeneous nature. In order to determine what part of Tycho Brahe's catalogue is homogeneous, we have built individual latitudinal error frequency histograms for each of the celestial areas *A, Zod A, B, Zod B, C, D* and *M*. See figs. 9.1-9.7 for results.

Bear in mind that the celestial areas in question have been defined above, in our analysis of the Almagest (see section 3 of Chapter 2). In order to build these histograms we have calculated the ecliptic stellar coordinates for the epoch of 1600 A.D. Then we compared the latitudes of the stars from Tycho Brahe's catalogue with the calculated latitudes of respective stars. In figs. 9.1-9.7 the error rate scale is divided into segments of 0.5' each. This scale is horizontal. What we find on the vertical is the manifestation frequency of a certain error rate.

The resulting histograms demonstrate that among the latitudinal errors in Tycho Brahe's catalogue coordinates we do indeed find rejects. If we are to presume that stellar coordinate measurement errors are distributed normally, which would be a justified expectation, we find that about 15% of error values are located outside the interval 3σ. These values are "rejects". Moreover, we notice that the histograms are shifted towards zero. The approximate value of this shift equals 2' and tells us that Tycho Brahe's catalogue contains a systematic error in stellar latitude with parameter $\gamma \approx 2'$. Remember that values γ and φ, which parameterize the systematic error of the catalogue, were introduced in Chapter 5.

The stars that we excluded from Tycho Brahe's catalogue in the course of reject filtration are the ones whose latitudinal error does not fit into normal distribution. This was done for each of celestial areas *A, B, C, D* and *M* individually. More precisely, we rejected the stars from areas *A, B* and *M* whose latitudinal discrepancy value was more than 5' or less than –7'. All stars with the absolute latitudinal discrepancy value greater than 5' were rejected from area *C*, as well as all area *D* stars with a discrepancy of either more than 4' or less than –3'. The indicated error boundaries have been estimated approximately, judging by figs. 9.1-9.7. We have rejected a total of 187 stars out of 1005. The quantity of the remaining stars (818) is close to 777, which is the amount of stars observed by Tycho Brahe himself, as the legend has it (see [65], page 126).

After the "reject filtering" of Tycho Brahe's catalogue as described above, systematic error parameters $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ were calculated by the remaining part of the catalogue as functions of presumed dating t. See Chapter 5 for respective definitions. The chosen t alteration interval begins with 1400 A.D., or $t = 5$, and ends with 1700 A.D., or with $t = 2$. The re-

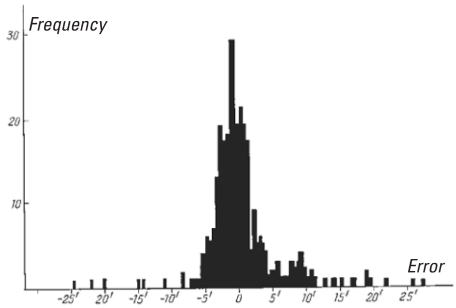

Fig. 9.1. Latitudinal discrepancy histogram for celestial region *A* in Tycho Brahe's catalogue, with $t = 3$.

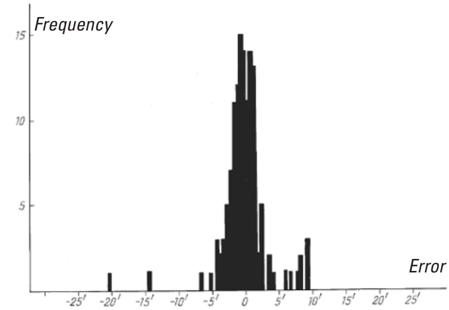

Fig. 9.2. Latitudinal discrepancy histogram for celestial region *Zod A* in Tycho Brahe's catalogue, with $t = 3$.

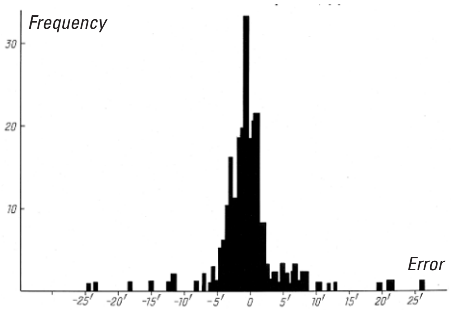

Fig. 9.3. Latitudinal discrepancy histogram for celestial region *B* in Tycho Brahe's catalogue, with $t = 3$.

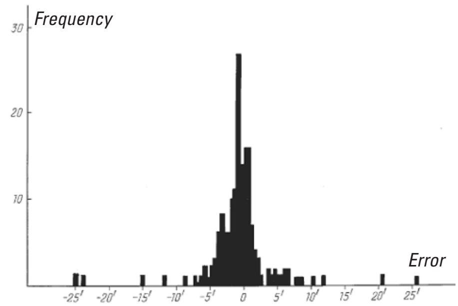

Fig. 9.4. Latitudinal discrepancy histogram for celestial region *Zod B* in Tycho Brahe's catalogue, with $t = 3$.

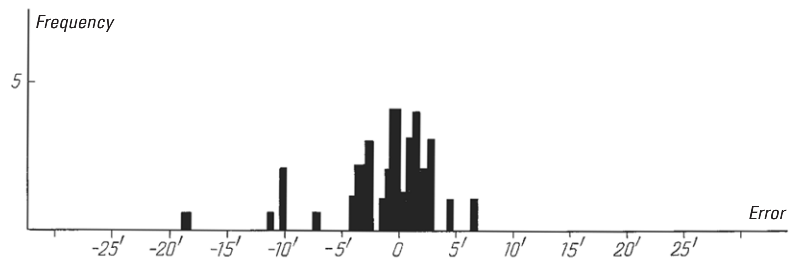

Fig. 9.5. Latitudinal discrepancy histogram for celestial region *C* in Tycho Brahe's catalogue, with $t = 3$.

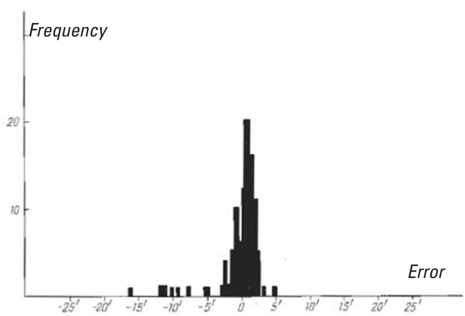

Fig. 9.6. Latitudinal discrepancy histogram for celestial region *D* in Tycho Brahe's catalogue, with $t = 3$.

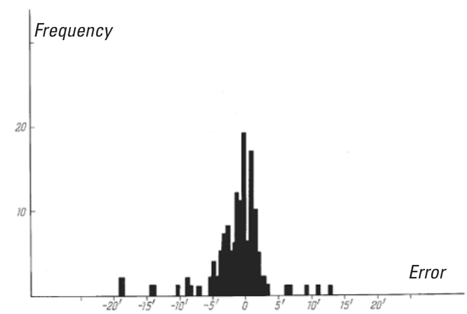

Fig. 9.7. Latitudinal discrepancy histogram for celestial region *M* in Tycho Brahe's catalogue, with $t = 3$.

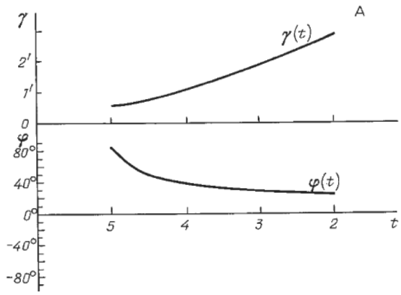

Fig. 9.8. The graphs of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ for celestial region *A* in Tycho Brahe's catalogue.

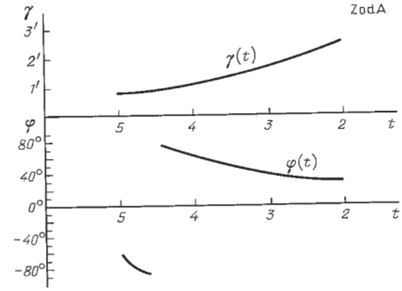

Fig. 9.9. The graphs of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ for celestial region *Zod A* in Tycho Brahe's catalogue.

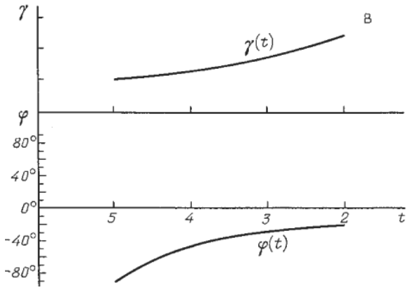

Fig. 9.10. The graphs of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ for celestial region *B* in Tycho Brahe's catalogue.

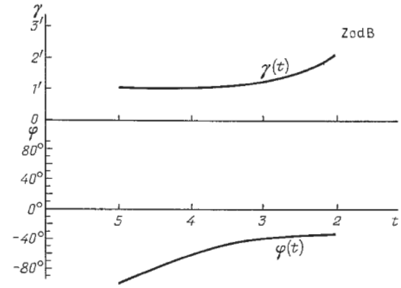

Fig. 9.11. The graphs of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ for celestial region *Zod B* in Tycho Brahe's catalogue.

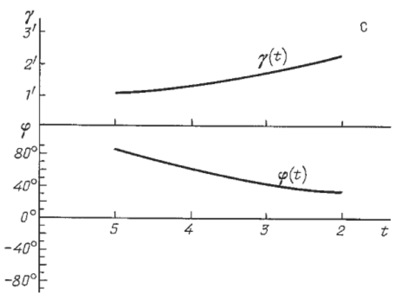

Fig. 9.12. The graphs of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ for celestial region *C* in Tycho Brahe's catalogue.

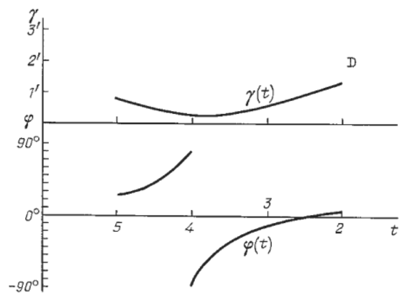

Fig. 9.13. The graphs of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ for celestial region *D* in Tycho Brahe's catalogue.

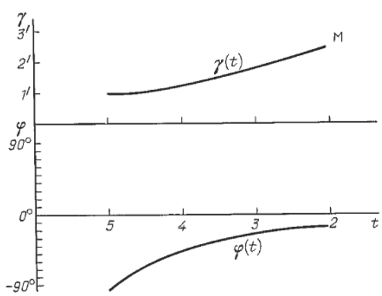

Fig. 9.14. The graphs of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ for celestial region *M* in Tycho Brahe's catalogue.

sult of calculating the functions of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ for each of the seven celestial regions (see section 2 of Chapter 6) is represented graphically in figs. 9.8-9.14. The graphs clearly demonstrate that parameter φ assumes substantially different values for different celestial areas in Tycho Brahe's catalogue, and doesn't appear to represent a systematic error. Parameter γ, on the other hand, behaves in the exact same manner for every celestial region.

A propos, we observed a similar situation in our analysis of the Almagest catalogue, qv in Chapter 6. The $\gamma_{stat}(t)$ graphs for celestial regions *A, Zod A, B, Zod B, C* and *M* from Tycho Brahe's catalogue resemble each other, qv in figs. 9.8-9.14. Celestial area *D* is the only exception here – parameter γ behaves differently for this area, qv in fig. 9.13. Therefore we rejected the stars from celestial region *D* in our dating of Tycho Brahe's observations.

2.3. The choice of the informative kernel for Tycho Brahe's catalogue

According to the astronomical observation dating algorithm that we suggest, we have to choose the informative kernel of Tycho Brahe's catalogue. As it is pointed out in [643] (see section 8 of the Introduction to [643]), Tycho Brahe chose 21 basis stars in the vicinity of the Zodiac, having estimated the equatorial coordinates of these stars with maximal possible precision. He would then convert them into ecliptic coordinates. The list of such stars was borrowed from [1049] (see table 9.1).

For constellations that contain stars from this list we have found group errors $\gamma^G_{stat}(t)$ and $\varphi^G_{stat}(t)$ for $t = 3$. See section 3 of Chapter 6 for definitions of these values. The stars from constellations whose group error $\gamma^G_{stat}(t)$ differed from $\gamma^{ZodA}_{stat}(t)$ by more than 2' for $t = 3$ were excluded from further consideration. For the remaining constellations we calculated the percentage of stars whose latitudinal error does nor exceed 1', 2' and 3' respectively for $t = 3$. We have then calculated the square average latitudinal discrepancy for each constellation – both disregarding and considering the systematic error, with parameters $\gamma = \gamma^G_{stat}(t)$ and $\varphi = \varphi^G_{stat}(t)$ for $t = 3$. The same parameters were calculated after the compensation of the common systematic error with parameters $\gamma = \gamma^{ZodA}_{stat}(3) = 1.8'$, $\varphi = 0$. It turns out that the compensation of the common systematic error leads us to the same result as the compensation of the group error for each of the constellations considered, qv in table 9.2. Now we can consider the systematic error to be *common* for the group of constellations that we have under study and use the values of $\gamma = \gamma^{ZodA}_{stat}(t)$, $\varphi = 0$.

We included 12 stars out of 21 into the informative kernel of Tycho Brahe's catalogue – the ones that remained in the catalogue after the "group error filtering" as described above. Apart from that, we included two fast and bright named stars into it – Arcturus = α Boo and Procyon = α CMi. The third fast named star (Sirius) was not included in the informative kernel, since it is located in celestial region *D* that possesses a unique systematic error, qv above. Therefore, the informative kernel of Tycho Brahe's catalogue consists of 14 stars:

γ Ari, α Ari = Hamal, ε Tau, α Tau = Aldebaran, γ Can = Aselli, γ Leo, α Leo = Regulus, γ Vir, α Vir = Spica, Δ Oph, α Aqu, α Pis, α Boo = Arcturus and α CMi = Procyon.

2.4. The dating of Tycho Brahe's observations

As it is implied by table 9.2, the residual square average latitudinal error after the compensation of the systematic compound with parameters $\gamma = \gamma^{ZodA}_{stat}(t)$, $\varphi = 0$ fluctuates within the boundaries of 1'-3' for the constellations that contain informative kernel stars. The percentage of stars in constellations whose latitudinal error is less than 2' is greater than 50% in all cases.

According to the dating interval suggested in Chapter 7, one has to take 2' as the Δ threshold. Then one would have to determine the time interval for which the latitudinal discrepancy of all the informative kernel stars does not exceed $\Delta = 2'$. The resultant interval shall contain possible datings of Tycho Brahe's observations.

We calculated this time interval. It begins with 1510 A.D. and ends in 1620 A.D. ($2.8 \le t \le 3.9$). We use a 10-year step for Tycho Brahe's catalogue. Here, as above, presumed catalogue dating *t* is measured in centuries and counted backwards from 1900.

	Base stars from Tycho Brahe's catalogue	α_{1900}, hours, minutes and seconds	β_{1900}, hours, minutes and seconds	Proper motion rate per annum, in arc seconds		l = ecliptic longitude	b = ecliptic latitude	Value
				V_α	V_δ			
		According to the modern catalogue ([1197])				According to Tycho Brahe's catalogue ([1024])		
1	5 γ Ari	1.48.02,4	+18°48'21"	+0.079	–0.108	Ari 27°37.0'	+7°08.5'	4
2	13 α Ari	2.01.32,0	+22°59'23"	+0.190	–0.144	Tau 2°06.0'	+9°57.0'	3
3	74 ε Tau	4.22.46,5	+18°57'31"	+0.108	–0.036	Gem 2°53.0'	–2°36.5'	3
4	87 α Tau	4.30.10,9	+16°18'30"	+0.065	–0.189	Gem 4°12.5'	–5°31.0'	1
5	13 μ Gem	6.16.54,6	+22°33'54"	+0.055	–0.112	Gem 29°44.0'	–0°53.0'	3
6	24 γ Gem	6.31.56,1	+16°29'05"	+0.043	–0.044	Can 3°31.0'	–6°48.5'	2
7	78 β Gem	7.39.11,8	+28°16'04"	–0.627	–0.051	Can 17°43.0'	+6°38.0'	2
8	43 γ Can	8.37.29,9	+21°49'42"	–0.103	–0.043	Leo 1°57.0'	+3°08.0'	4
9	41 γ Leo	10.14.27,6	+20°20'51"	+0.307	–0.151	Leo 23°59.0'	+8°47.0'	2
10	32 α Leo	10.03.02,8	+12°27'22"	–0.249	–0.003	Leo 24°17.0'	+0°26.5'	1
11	29 γ Vir	12.36.35,5	–0°54'03"	–0.568	–0.008	Lib 4°35.5'	+2°50.0'	3
12	67 α Vir	13.19.55,4	–10°38'22"	–0.043	–0.033	Lib 18°16.0'	–1°59.0'	1
13	27 β Lib	15.11.37,4	–9°00'50"	–0.098	–0.023	Vir 13°48.0'	+8°35.0'	2
14	1 δ Oph	16.19.06,2	–3°26'13"	–0.048	–0.145	Vir 26°44.5'	+17°19.0'	3
15	21 α Sco	16.23.16,4	–26°13'26"	–0.007	–0.023	Sag 4°13.0	–4°27.0'	1
16	39 o Sag	18.58.41,4	–21°53'17"	+0.079	–0.060	Cap 9°28.0'	+0°59.0'	4
17	53 α Aqi	19.45.54,2	+8°36'15"	+0.537	+0.385	Cap 26°09.0'	+29°21.5'	2
18	40 γ Capr	21.34.33,1	–17°06'51"	+0.188	–0.022	Aqu 16°14.0'	–2°26.0'	3
19	22 β Aqu	21.26.17,7	–6°00'40"	+0.019	–0.005	Aqu 17°51.0'	+8°42.0'	3
20	54 α Peg	22.59.46,7	+14°40'02"	+0.062	–0.038	Pis 17°56.5'	+19°26.0'	2

Table 9.1. The base stars of Tycho Brahe's catalogue.

The behaviour of the maximal latitudinal error for the stars of the informative kernel with t varying from 2.6 to 4.2 is illustrated by a series of drawings similar to fig. 7.10 illustrating the Almagest example (see fig. 9.15).

Parameter area (γ, φ) with solid black shading has the maximal latitudinal error of 2'. The area with regular shading has the error maximum of 2.5'. Fig. 9.15 demonstrates that raising the threshold to the level of 2.5' expands the possible dating interval to 1490-1640 A.D. and not more (instead of the former years 1510-1620 A.D.) If we chose a level of Δ = 3', we would come up with a possible dating interval of 1480-1620 A.D.

Therefore, as is the case with the Almagest catalogue, the boundaries of the possible dating interval for Tycho Brahe's catalogue are only marginally dependent on the level variation of Δ.

Additional calculations demonstrated that the dating interval of Tycho Brahe's observations is also stable in cases of informative kernel contingent variation.

2.5. Conclusions

1) Our method as applied to Tycho Brahe's catalogue yields a possible dating interval of 110 years (between 1510 and 1601 A.D.) The resulting interval covers the lifetime of Tycho Brahe (1546-1601). The period of Tycho Brahe's observations in the observatory of Uraniborg (1576-1597) locates in the middle of this period, or around 1565.

Constellation. Number of stars in a constellation	*Turn of the celestial sphere*	*The percentage of stars in a constellation whose latitudinal error rate doesn't exceed the value of:*			*Residual square average discrepancy* $\hat{\sigma}$
		1′	*2′*	*3′*	
Cancer, 13 stars	- (condition before the turn)	38	77	77	2.40'
	optimal for *Zod A*	61	85	92	2.37'
	optimal for constellation	61	77	92	2.37'
	$\gamma = \gamma_{stat}^{ZodA}(t),\ \varphi = 0$	46	77	92	2.37'
Leo, 36 stars	- (condition before the turn)	61	83	94	1.41'
	optimal for *Zod A*	55	80	94	1.44'
	optimal for constellation	61	83	94	1.35'
	$\gamma = \gamma_{stat}^{ZodA}(t),\ \varphi = 0$	47	75	94	1.63'
Taurus, 37 stars	- (condition before the turn)	76	89	94	1.18'
	optimal for *Zod A*	54	92	97	1.31'
	optimal for constellation	67	92	94	1.17'
	$\gamma = \gamma_{stat}^{ZodA}(t),\ \varphi = 0$	24	62	94	1.94'
Pisces, 31 stars	- (condition before the turn)	61	77	90	1.81'
	optimal for *Zod A*	48	81	90	1.97'
	optimal for constellation	64	81	90	1.79'
	$\gamma = \gamma_{stat}^{ZodA}(t),\ \varphi = 0$	45	77	87	1.87'
Aquarius, 34 stars	- (condition before the turn)	29	56	76	2.49'
	optimal for *Zod A*	32	59	82	2.23'
	optimal for constellation	35	82	91	1.63'
	$\gamma = \gamma_{stat}^{ZodA}(t),\ \varphi = 0$	38	65	91	1.90'
Virgo, 32 stars	- (condition before the turn)	25	72	94	1.80'
	optimal for *Zod A*	34	72	94	1.83'
	optimal for constellation	62	91	100	1.16'
	$\gamma = \gamma_{stat}^{ZodA}(t),\ \varphi = 0$	59	91	94	1.22'
Aries, 20 stars	– (condition before the turn)	65	85	100	1.22'
	optimal for *Zod A*	60	40	100	1.21'
	optimal for constellation	50	95	100	1.20'
	$\gamma = \gamma_{stat}^{ZodA}(t),\ \varphi = 0$	45	65	90	1.63'
Ophiuchus, 24 stars	- (condition before the turn)	17	37	70	2.84'
	optimal for *Zod A*	46	79	92	1.93'
	optimal for constellation	50	92	92	1.69'
	$\gamma = \gamma_{stat}^{ZodA}(t),\ \varphi = 0$	25	54	83	2.40'

Table 9.2. Calculation results for Tycho Brahe's catalogue.

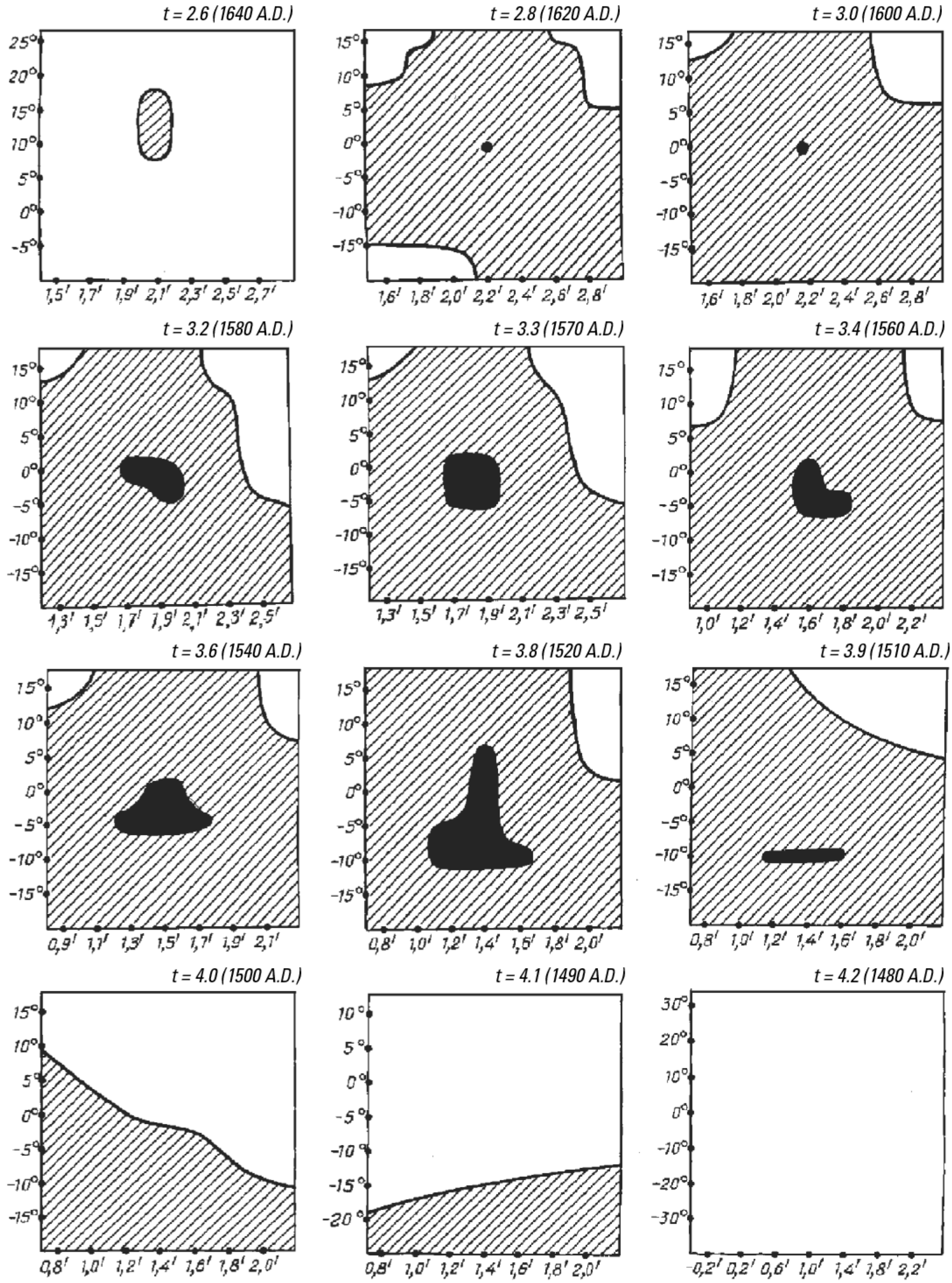

Fig. 9.15. Maximal latitudinal discrepancy $\Delta(t, \gamma, \varphi)$ for Tycho Brahe's catalogue, for t values ranging between 2.6 and 4.2, or 1480 A.D. to 1640 A.D. Area with Δ no more than 2' is shaded black; area with Δ no more than 2'30" has regular shading.

2) Possible dating interval of Tycho Brahe's observation demonstrates a good level of stability under variations of Δ level as well as variations in the informative kernel contingent. Raising the Δ level from 2' to 3' makes this interval grow to 200 years (1480-1680 A.D.)

3) The resulting possible dating interval equalling 110 years is roughly 6 times shorter than the one calculated for the Almagest (700 years). This corresponds to the fact that Tycho Brahe's catalogue is 5-6 times more precise than the Almagest – namely, it has an error threshold of 2'-3' as opposed to 10'-15'.

4) The statistical possible dating interval of Tycho Brahe's catalogue correlates with the geometrical interval for trust levels of $1 - \varepsilon > 0.9$.

3. ULUGBEK'S CATALOGUE

3.1. A general characteristic of Ulugbek's catalogue and its dating result

Ulugbek's catalogue is presumed to be a more precise version of the Almagest star catalogue based on the astronomical observations performed in the observatory of Samarkand in the middle of the XV century A.D., in the reign of king Ulugbek ([1339]). However, according to Peters and Knobel, "although Ulugbek did in fact compile a more precise catalogue of Ptolemaic stars, this catalogue never became widely-used" ([1339], page 7). A study of Ulugbek's catalogue demonstrates that it is de facto a catalogue of Ptolemaic stars. It isn't just the stellar contingent that coincides for both catalogues, but also the order of stars as listed in Ulugbek's catalogue and the Almagest, exceptions being few and far between. There are 1019 stars in Ulugbek's catalogue. Ecliptical coordinate values are given to the minute, yet the real precision of this catalogue is substantially lower. Some researchers estimated it to equal 3'-5' (see [65]). However, our calculations demonstrate the residual dispersion of the latitudinal error in Ulugbek's catalogue to equal 16.5' for celestial area *Zod A*, which is where we find the catalogue at its most precise. Thus, the real latitudinal precision of Ulugbek's catalogue is about 30'-35', which is lower than that of the Almagest to a great extent!

On the other hand, systematic error γ is smaller in Ulugbek's catalogue than in the Almagest. As a result, latitudinal precision of the former in its initial form, or prior to the exclusion of the systematic error, is somewhat higher than the latitudinal precision in the original text of the Almagest catalogue. The difference equals 5'-6'. However, this difference is rather insubstantial when compared to the rate of the (latitudinal) error in both catalogues taken in their initial form, without the compensation of the systematic error. It is hardly surprising that Ulugbek's catalogue never replaced the Almagest in scientific circulations. In fig. 9.15a we cite the title page from Ulugbek's catalogue.

The histogram of Ulugbek catalogue's latitudinal error rate for the stars from celestial area *A* can be seen in fig. 9.16. Before the histogram was built, all the stars whose latitudinal discrepancy exceeded 1 degree for $t = 5$, or 1400 A.D., were excluded from consideration.

Our calculations also demonstrate that Ulugbek's catalogue contains outright borrowings from the Almagest (or vice versa). In fig. 9.17 we see a difference histogram between stellar latitudes in Ulugbek's catalogue and the latitudes of the respective stars in

جداول مواضع ثوابت در طول وعرض

كه بر صد يافته است الغ بيك بن شاهرخ بن تيمور

SIVE

TABVLÆ

LONG. AC LAT. STELLARUM FIXARVM,

EX OBSERVATIONE

ULUGH BEIGHI,

TAMERLANIS Magni Nepotis, Regionum ultra citráque GJIHUN (i. Oxum) Principis potentissimi.

Ex tribus invicem collatis MSS Persicis jam primùm Luce ac Latio donavit, & Commentariis illustravit THOMAS HYDE A. M. e Coll. Reginæ Oxon.

In Calce Libri accesserunt MOHAMMEDIS TIZINI Tabulæ Declinationum & Rectarum Ascensionum. Additur demum Elenchus Nominum Stellarum.

OXONII

Typis Henrici Hall Academiæ Typographi Sumptibus Authoris. Venales prostant apud Richardum Davis Bibliopolam. CIↃIↃCLXV

Fig. 9.15a. Title page of Ulugbek's catalogue.

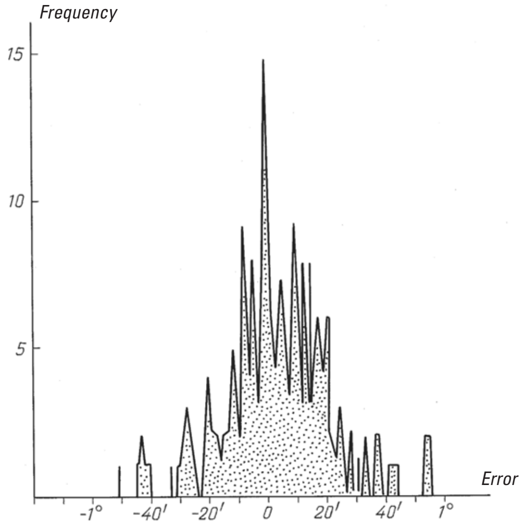

Fig. 9.16. Latitudinal discrepancy histogram for celestial area *Zod A* in Ulugbek's catalogue, with $t = 5$.

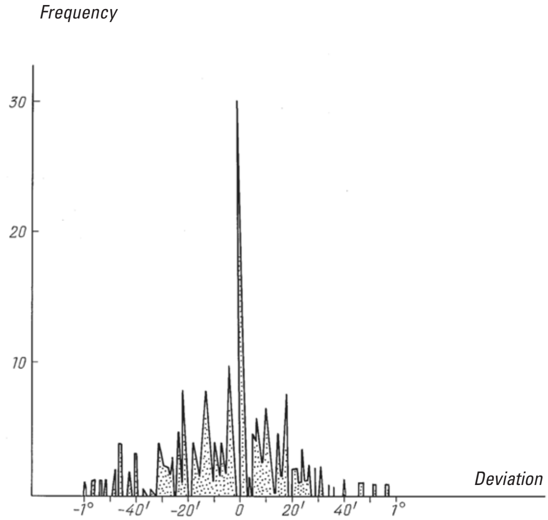

Fig. 9.17. Difference frequency histogram for stellar latitudes from Ulugbek's catalogue and the Almagest, without systematic error compensation (Ulugbek – Almagest).

the Almagest. Identifying Ulugbek's stars as their Almagest counterparts presents no problems since, as it has been pointed out, the order of stars coincides for both catalogues.

The abrupt peak at zero in fig. 9.17 corresponds to the group of stars whose latitudes coincide completely in both catalogues. This peak is great enough to leave no room for speculation about its being of a random character.

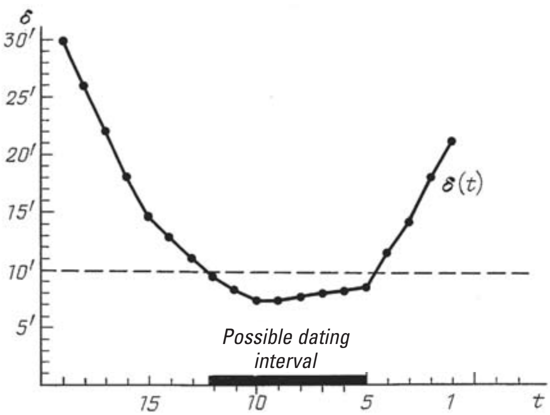

Fig. 9.18. Minimal discrepancy graph of $\Delta(t)$ for the stars from the informative kernel of Ulugbek's catalogue, depending on the presumed dating t.

3.2. Systematic errors in Ulugbek's catalogue

Parameters of the systematic error $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ were calculated for celestial region *Zod A* from Ulugbek's catalogue with the alleged datings ranging from 100 B.C. and 1800 A.D. ($1 \le t \le 20$). See section 2 of Chapter 6 for more details concerning the calculation of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$. The results of $\gamma_{stat}(t)$ and $\varphi_{stat}(t)$ computations for the three presumed datings of 1500 A.D. ($t = 4$), 900 A.D. ($t = 10$) and 400 A.D. ($t = 15$) are compiled in table 9.3, which is where we also find the square average error values of $\hat{\sigma}$ before and after the compensation of the systematic error with parameters $\gamma = \gamma_{stat}$ and $\varphi = \varphi_{stat}$.

3.3. The choice of the informative kernel and the Δ threshold. The dating of Ulugbek's catalogue

Let us compile the informative kernel of Ulugbek's catalogue using named stars from area *A* as the most thoroughly observed part of the sky and its immediate vicinity, just as we did in the dating of the Almagest catalogue. We shall come up with the same 9 stars from area A as we find named in the Almagest, i. e.:

Arcturus = α Boo, Regulus = α Leo, Spica = α Vir, Antares = α Sco, Capella = α Aur, Lyra = Vega = α Lyr, Aselli = α Can, Procyon = α CMi and Previndemiatrix = ε Vir.

This time we do not exclude the star Previndemiatrix from consideration the way we did in case of the Almagest, since its coordinates in Ulugbek's catalogue aren't a result of later calculations, and hence appear to contain no scribe errors ([1024]).

According to table 9.3, we must choose 10' as the value of precision threshold Δ for the latitudes of named stars from celestial area *A*, as we have done in case of the Almagest. Indeed, the mean-square latitudinal discrepancy for celestial area *Zod A* equals 16.5' after the compensation of the systematic error. 45% of the stars from this area have a residual latitudinal error of 10' maximum after the compensation of the systematic error.

Having selected the informative kernel of the catalogue and set the 10' Δ threshold, we get the geometrical interval of possible datings for Ulugbek's catalogue, namely, 700 A.D. – 1450 A.D. The statistical interval of possible datings coincides with the geometrical with a trust level of higher than 0.4. The resultant possible dating interval of Ulugbek's catalogue remains stable when the level of Δ changes, as well as in case varying informative kernel contingent. Thus, for Δ = 15' this interval expands to 400 A.D. – 1600 A.D.

The minimal latitudinal discrepancy graph $\Delta(t)$ for the informative kernel stars is built in fig. 9.18 as a function of the alleged dating *t*. This graph is similar to the one we find in fig. 7.27 as calculated for the Almagest catalogue. Bear in mind that $\Delta(t)$ is the minimum for all possible methods of making the stellar configuration of the informative kernel of Ulugbek's catalogue correspond with the real (calculated) stellar configuration for maximal latitudinal error time moment *t* involving all the stars of the informative kernel. It is obvious that if one fixes the method of superimposing two stellar combinations over each other, one can calculate the latitudinal discrepancy for each star individually and then take the maximal value of this error for all the stars of the configuration. Fig. 9.18 demonstrates in particular the possible dating interval variations of Ulugbek's catalogue that result from the variation of level Δ. A comparison of figs. 9.18 and 7.27 confirms the circumstance that we pointed out above, namely, the fact that the coordinate precision characteristics of both the Almagest and Ulugbek's catalogue are similar to one another.

3.4. Conclusions

1) The geometrical possible dating interval of Ulugbek's catalogue begins in 700 A.D. and ends in 1450 A.D. It covers the Scaligerian dating of the catalogue's creation, which is 1437 A.D., although we observe this dating to be shifted towards the very end of the calculated interval. On the other hand, this interval is remarkably similar to the one we came up with for the Almagest – 600 A.D. to 1300 A.D. It is therefore possible that both catalogues were compiled around the same time.

2) Precision characteristics of Ulugbek's and Ptolemy's catalogues virtually coincide. The systematic compound of the latitudinal error is greater in the Almagest as compared to Ulugbek's catalogue – approximately 20' instead of 10'. The residual random compound for celestial area *Zod A* is, on the other hand, somewhat greater in Ulugbek's catalogue, namely, $\hat{\sigma}$ = 16.5' instead of 12.8'. It was also discovered that the coordinates of 48 stars present in both catalogues coincide completely, which is a result of one catalogue borrowing from the other.

3) The possible dating interval of Ulugbek's catalogue is stable to Δ level changes as well as variations of the informative kernel contingent.

Dates	γ_{stat}	φ_{stat}	σ_{init}	σ_{min}
t = 4, or 1500 A.D.	11.55	–43°	18.36	16.43
t = 10, or 900 A.D.	10.33	–60°	17.92	16.33
t = 15, or 400 A.D.	10.87	–76°	18.1	16.35

Table 9.3. Ulugbek's catalogue. Calculation results $\gamma_{stat}(t)$, $\varphi_{stat}(t)$ for the three presumed datings of 1500 A.D., 900 A.D. and 400 A.D.

4) The statistical possible dating interval of Ulugbek's catalogue coincides with the geometrical interval for any trust level $1 - \varepsilon > 0.4$. If we raise the threshold of Δ to 15', the corresponding statistical interval for $1 - \varepsilon \leq 0.999$ is narrowed to roughly 100 years off the top boundary, reaching up to 1500 A.D. instead of 1600 A.D.

4. THE CATALOGUE OF HEVELIUS

4.1. The dependency between the catalogues of Tycho Brahe and Hevelius

The catalogue of Hevelius was compiled in the second half of the XVII century, already after the invention of the telescope. However, Hevelius was reluctant to use the telescope, considering his naked eye observations to be more precise ([1024]). This was confirmed by Galley after a "competition" of sorts that he entered with Hevelius when they were observing the coordinates of the same stars using different methods – the telescope for Galley and traditional astronomical instruments for Hevelius. The results differed by a mere 1" ([1024]). Literature of the subsequent epochs adhered to the opinion that Hevelius was just as precise in his observations as the astronomers who used telescopes (1-second precision rate). Stellar coordinates in the catalogue of Hevelius are given with arc seconds.

Our analysis does not confirm this popular point of view. We have studied several configurations comprising bright named stars from the catalogue of Hevelius, among which there were three fast stars – Arcturus = α Boo, Sirius – α CMa and Procyon = α CMi. Values of t from the interval of $1 \leq t \leq 5$, or 1400 A.D. – 1800 A.D. were chosen to represent the presumed dating of Hevelius' observations. Moreover, what we tried to find every time was such a superimposition of the stellar configuration from the catalogue of Hevelius over the respective real (calculated) stellar configuration for time moment t for which the maximal latitudinal discrepancy for the configuration stars would be as low as possible. Under "latitudes" we understand the ecliptic latitudes of stars, as usual.

We found out that the celestial sphere rotation parameters that define this optimal superimposition equal zero ($\gamma = 0$, $\varphi = 0$). The implication should be

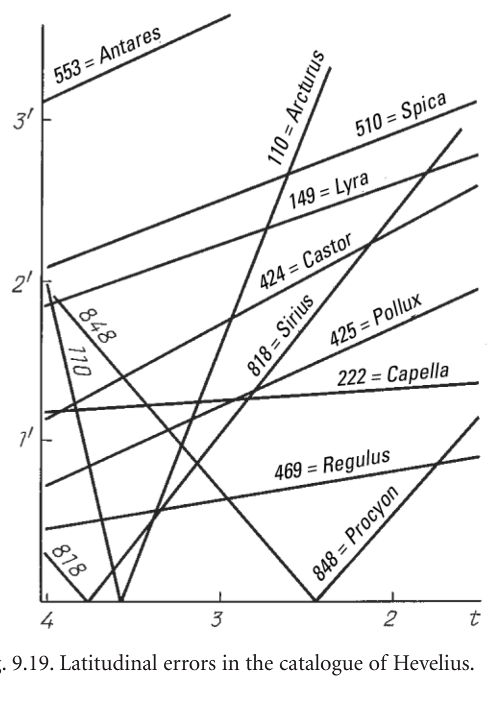

Fig. 9.19. Latitudinal errors in the catalogue of Hevelius.

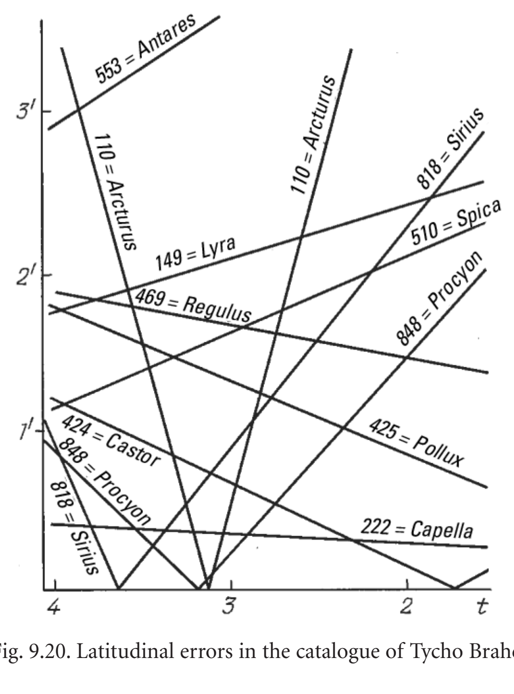

Fig. 9.20. Latitudinal errors in the catalogue of Tycho Brahe.

that the stellar configurations from the catalogue of Hevelius that we studied contain no systematic error, or that there is no shift across the sphere discovered in their coordinates according to Hevelius, which would make the systematic error equal zero. However, random latitudinal errors have the same average rate as those contained in the catalogue of Tycho Brahe, namely, 2'-3'. All of this considering how the scale grade value in the catalogue of Hevelius is 60 times smaller than that of Tycho Brahe's catalogue – 1" instead of 1'. It turns out that the latitudinal errors made by Hevelius are 100-200 times greater than the grade value of his numerical scale!

This circumstance is illustrated in fig. 9.19. It contains the latitudinal error graphs as functions of presumed dating *t* for each of 10 named bright stars from the catalogue of Hevelius:

Arcturus = α Boo, Sirius = α CMa, Procyon = α CMi, Antares = α Sco, Vega = Lyra = α Lyr, Pollux = β Gem, Castor = α Gem, Spica = α Vir, Capella = α Aur and Regulus = α Leo.

In fig. 9.20 we see the same graph built for the catalogue of Tycho Brahe. A comparison of figs. 9.19 and 9.20 demonstrates the latitudinal error to be the same for both catalogues. Furthermore, actual error values for some of the stars contained in the catalogues of Tycho Brahe and Hevelius are close to each other. This applies to Arcturus, Sirius, Antares, Procyon and Lyra = Vega. This is a clear indication of a dependency between the catalogues of Tycho Brahe and Hevelius.

4.2. Conclusions

1) The precision of Hevelius' catalogue is hardly any higher than that of Tycho Brahe's catalogue. This observation is a result of the analysis of bright named star configurations in the catalogue of Hevelius.

2) The catalogue of Hevelius is apparently dependent on the catalogue of Tycho Brahe. This dependency is most obviously manifest for the group of fast bright stars, namely, Arcturus, Sirius and Procyon. As the fast named stars comprise the suggested dating basis of the old star catalogues, the independent dating of Hevelius' catalogues makes no sense at all. The result shall be close to the one we got for Tycho Brahe's catalogue.

5. THE CATALOGUE OF AL-SUFI

We borrowed the star catalogue of Al-Sufi from [1394]. It is usually presumed that the catalogue of Al-Sufi was compiled by the latter from his own observations ([516]). The author opposes himself to the astronomers who use cosmospheres and ready-made catalogues such as the Almagest instead of actual star observations when they compile catalogues under their own names.

He tells us the following:

"I have seen many of those who strive after the knowledge of immobile stars… and discovered them to be people of two categories.

The first category follows the method of the astronomers and uses cosmospheres painted by artists who know not the stars and use the longitudes and the latitudes that they find in books in order to mark the stellar location upon the sphere, unable to tell the truth from the errors. Afterwards knowing people study the spheres and see that the stars drawn thereupon differ from the ones observed in the sky. The makers of cosmospheres make references to astronomical tables whose authors claim to have observed the stars and estimated their positions themselves. In reality, they merely chose the most famous of the stars, the ones known to all such as the Eye of the Bull, the Heart of the Lion [Regulus – Auth.], Virgin's Ear of Wheat [Spica – Auth.], the three stars in the forehead of the Scorpion as well as the heart of the latter [Antares – Auth.] – the very stars whose longitudes and latitudes Ptolemy says to have observed and included in the Almagest, since all of these stars are close to the ecliptic. As for the other stars that Ptolemy indicates in the star catalogue of his book, they would add whichever value they fancied to each one of them. Having shifted these stars in space by the value of the interval between their own lifetimes and that of Ptolemy, they would add several minutes to Ptolemy's longitudes or subtract them from the latter to make the impression that the observations were conducted by themselves alone, and that the process yielded some individual differences in the longitudes and the latitudes regardless of either the general stellar increments or the amount of time that separates them from Ptolemy. All of this was done with no actual knowledge of the stars. Such are Al-Batani, Atarid and others.

I have carefully studied many copies of the Almagest and found that they differ from the multitude of immobile stars. The second category of people who seek the knowledge of immobile stars consists of amateurs". Quoting according to [544], Volume 4, pages 239-241.

However, the comparison of the stellar coordinates from the Almagest and Al-Sufi's catalogue makes it obvious that the catalogue of Al-Sufi is but one of the numerous existing versions of the Almagest.

Indeed, the order in which the stars are listed in both the Almagest and Al-Sufi's catalogue is exactly the same. The longitudes of all the stars as given by Al-Sufi are made greater with a shift of 12°42' as compared to the Almagest catalogue in its canonical version ([1339]), and the latitudes are exactly the same as in the latter. Let us point out that the shift of longitudes by a single constant, or rendering them to another historical epoch by precession, is indeed present in some of handwritten and printed copies of the Almagest – manuscript 11 from the copy cited in [1339], for instance. This so-called "Venetian Codex 312" contains stellar latitudes 17 degrees greater than Ptolemy's ([1339], page 20).

Peters and Knobel comment as follows: "One sees that the true [according to Peters and Knobel – Auth.] longitudes of Ptolemy, as well as the modified variety, replaced the original figures" ([1339]), page 20. One way or another, what we encounter here qualifies as traces of certain "activities" involving the Almagest catalogue. We see that the longitudes of the Almagest catalogue were shifted into various historical epochs for some reason. Later editors of the Almagest may have initially been of different opinions on what longitudinal shift the catalogue required exactly, and subsequently agreed upon choosing the epoch of the

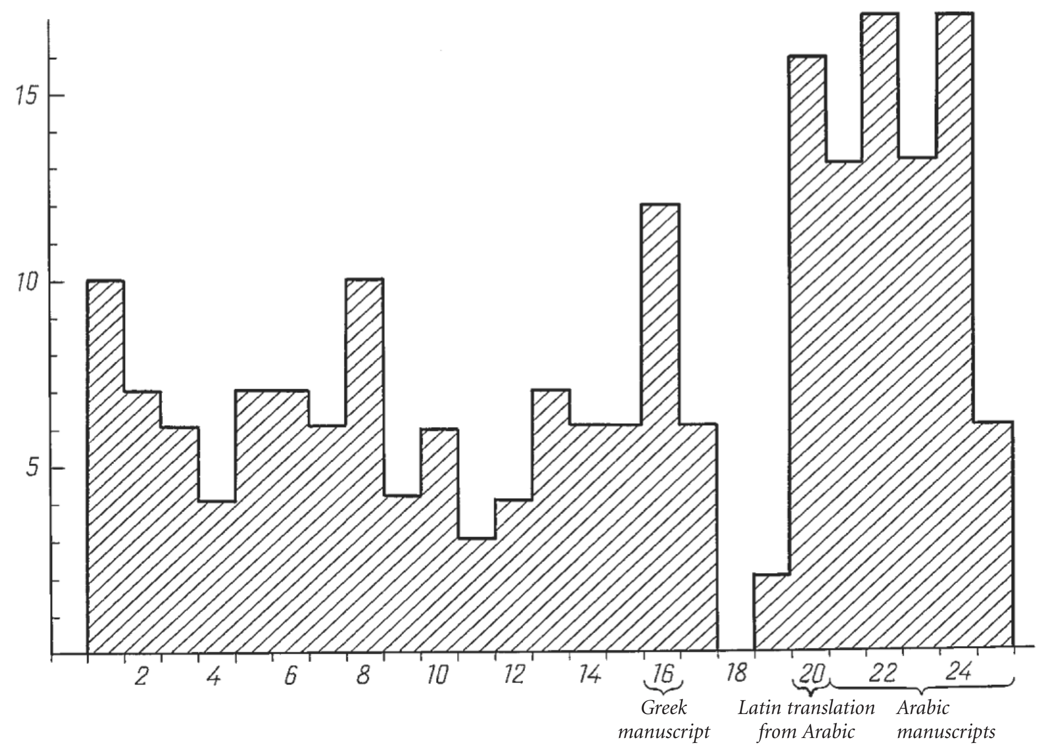

Fig. 9.21. The graph contains the following indications for each of the 25 Almagest manuscripts: the number of cases for which the discrepancy between the latitudes specified by Al-Sufi and the ones in the canonical version of the Almagest equals that of the manuscript under study.

very dawn of the new era. Studying the surviving copies of the Almagest critically in this light would indeed be of value to our research.

Furthermore, it turns out that in the Latin manuscript of the Almagest dating to the alleged year 1490 A.D. became transformed in the following way: "Observing the precession, the scribe added [to the star catalogue – Auth.] the stellar longitudes for the epoch of Adam, having set them to 3496 B.C. and rendered said longitudes to mid-XV century A.D." ([1017]:1), inset between pages 128 and 129. Thus, a Scaligerite historian may well date the Almagest to the antediluvian epoch of Adam – quite erroneously so.

We see yet another longitudinal precision shift of the Almagest catalogue into the epoch of the XVI century A.D. in the Latin edition of the Almagest that dates from 1537 (kept in Cologne; see more about it in Chapter 11).

A comparison of latitudes of all the stars contained in Al-Sufi's catalogue ([1394]) and the canonical version of the Almagest demonstrates that only 53 stars out of 1028 demonstrate differences in latitudes – a very typical rate for different copies of the Almagest. Furthermore, the latitudes for 35 out of these 53 stars of Al-Sufi's coincide with the versions of latitudes contained in the copies of the Almagest studied by Peters and Knobel ([1339]). Thus, the catalogue of Al-Sufi is merely a copy of the Almagest catalogue (we must point out that this conclusion was also made by the astronomer J. Evans ([1119] and [1120]), whose approach was an altogether different one).

In fig. 9.21 one sees the diagram indicating all cases for which the latitudes differing from the canonical version of the Almagest in Al-Sufi's catalogue coincide with those contained in one of the 25 Almagest manuscripts studied by Peters and Knobel in [1339]. The group of handwritten copies of the Almagest which Al-Sufi's catalogue resembles the most is numbered 20-24 in fig. 9.21. It is noteworthy that this group consists of Arabic manuscripts descended from the same prototype – the so-called "translation of Al-Mamon", or the translation of the Almagest that is presumed to have been made by Al-Mamon in the IX century A.D. (see [1339], page 23). Apparently, the catalogue of Al-Sufi contained in [1394] has to be attributed to the same group of Almagest copies.

Let us cite the conclusion made by Peters and Knobel: "Skjellerup's French translation of the Arabic catalogue by Abd Al-Rahman Al-Sufi is merely a version of Ptolemy's catalogue rendered to a different epoch" ([1339], page 7).

Nevertheless, historians carry on claiming Al-Sufi's catalogue to be of an independent nature for some bizarre reason and based on Al-Sufi's own observations which the venerable scholars of history declare to have "pursued the goal of verifying the star catalogues of Ptolemy and the astronomers of the Orient, correcting them according to empirical observation data" ([515], page 190).

We have thus witnessed the star catalogue of the Almagest to have been rendered to various "desired epochs" by different astronomers who used the longitudinal precession method, adding or subtracting some constant value. This could be done for a great variety of reasons. The resulting catalogue could become attributed to a different astronomer – Al-Sufi, for instance. In other cases Ptolemy's name and authorship were kept intact, but the "ancient" Ptolemy himself travelled backwards in time and wound up somewhere around the beginning of the new era due to the "indisputable proof" presented by the longitudes of his catalogue which were magically transformed into "ancient" by proxy of a simple arithmetical operation.