(it is easy to verify that $L_2(\Delta_r, \Delta_s) = L_2(\Delta_s, \Delta_r)$).

The quantity $L_2(\Delta_r, \Delta_s)$ is in no way related to the common names in Δ_r and Δ_s; they are not involved in its definition. However, the conditional frequency histograms for $L_2(\Delta_r, \Delta_s)$ for the lists Π and N calculated with the fixed value of $O(\Delta_r, \Delta_s)$ show that the dependence of L_2 on $O(\Delta_r, \Delta_s)$ is the same as that of L_0 on $O(\Delta_r, \Delta_s)$. The same is valid for $L_1(\Delta_r, \Delta_s)$, which signifies that a certain common factor leading to their statistical dependence is at the foundation of two outwardly unrelated quantities $L_2(\Delta_r, \Delta_s)$ and $O(\Delta_r, \Delta_s)$. It is clear that the availability of common duplicates is a factor of this kind. Hence, the discovered dependence speaks for the hypothesis regarding the existence of duplicates in Π and N.

The relation matrices for Π and N constructed by means of L_0, L_1 or L_2, respectively, turned out to lead to the same conclusion, i.e., to distinguish the same duplicate systems. Therefore, we shall sometimes simply write $L(\Delta_r, \Delta_s)$, meaning one of their three relations L_0, L_1 or L_2.

Note the difference between the relation matrices constructed by means of $L(\Delta_r, \Delta_s)$ and that derived from the common names for Π and N, viz., that the former yield a more complete and "purer" picture. In particular, if the value of $O(\Delta_r, \Delta_s)$ is large, then, as a rule, $L(\Delta_r, \Delta_s)$ is large; however, the converse is not valid.

The thresholds separating strong relations (which should lead to the conclusion regarding the dependence of neighbourhoods) from the weak ones (the conclusion being that the neighbourhoods are independent) were chosen in accordance with the magnitude of $O(\Delta_r, \Delta_s)$ as follows: the relation conditional frequency histograms were constructed from the matrix $a_{rs} = L(\Delta_r, \Delta_s)$ with the number of common names $O(\Delta_r, \Delta_s)$ being fixed. For the lists Π and N, all these histograms were of the form as in Fig. 84. The smallest values taken as the thresholds were to the right of which the histogram was vanishing. The relations exceeding the threshold value are called below essential. Note that all the intersecting neighbourhoods for the Π and N, as expected, turned out to be dependent according to the constructed test (i.e., their relations were essential).

14. Results related to the name list of Roman popes. Chronological shifts. Here and in the next items, we consider the consequences of the study of the relation matrix for the popes' lists, from Peter until 1950, and the Roman kings' and emperors' list from the 8th c. B.C. (starting from the 4th c. A.D., we mean here the Western Roman Empire) until the Holy Roman Empire in 962–1254 and the Hapsburg Empire in 1273–1619 A.D., the emperors' list extended up to 1700 A.D. (up to Leopold). To make the discussion of the results independent of the above argument, we recall the basic ideas of the method.

The so-called *relation matrix* is constructed from a large chronological list of rulers' names, for which each pair of connected fixed-length pieces (neighbourhoods) is associated with a number (relation), so that the following conditions are fulfilled, viz., in the case where the given list contains no duplicates and consists of a random (in a sense) name sequence, the mean value of the relation does not depend on the choice of the numbers of the neighbourhood pairs, and, in the case where the list does contain duplicates, the relation of the pairs possessing duplicating fibers is, in general, greater than for those without such fibers.